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Article

Mowing the grass

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Abstract

Mowing the grass is a cyclical pattern in counterterrorism campaigns where governments attack to destroy terrorist capacity, thereby achieving a period of quiet as groups recover. If groups expect their capacity to be destroyed, why build their capabilities in the first place? I analyze an infinite-horizon dynamic game where a group endogenously builds capacity in the face of potential attacks and capacity is an evolving, persistent variable. The model highlights that terrorist groups and governments have incentives to create strategic uncertainty and thus explains attack cycles without punishment strategies, revenge preferences or imperfect/incomplete information. I calibrate the model to time-series data in the Israeli–Palestinian conflict describing rockets fired from Gaza. The results illustrate a peace-making dilemma: altering the government's incentives will have comparatively minimal effects on long-term conflict dynamics, whereas changing the terrorists' incentives to acquire capacity would either increase the frequency of high-capacity terrorism or government attacks.

Keywords

Counterterrorism, group capacity, Israeli–Palestinian conflict, model calibration, terrorism

1. Introduction

Mowing the grass refers to a cyclical pattern in counterterrorism campaigns: the terrorist group builds its capacity for violence, and once it becomes too strong, the government attacks to destroy this capacity. Eventually the group rebuilds, and the process repeats. The phrase is associated with the Israeli-Palestinian conflict after the Israeli Defense Force (IDF) launched *Operation Protective Edge*, a military incursion into the Gaza Strip aimed at stopping Hamas rocket fire into Israel (Inbar and Shamir, 2014). The pattern also appears more broadly. The Obama and Trump administrations used the

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phrase to describe U.S. operations against terrorist organizations in Africa and Asia (Snow, 2019; Bowden, 2021).

When both the government and the terrorist group comprehend each other's behavior and the larger fundamentals of conflict, mowing the grass highlights a strategic tension. If terrorist groups anticipate the attacks that would arise if they increase their capacity, why build up capabilities that will only be destroyed? In contrast, mowing the grass is suppose to be a cost-effective counterterrorism strategy, where government attacks generate a period of extended peace. If the government anticipates that the group will immediately rebuild, why continue to attack when it means exerting costly effort to attack again?

In this article, I study this strategic environment using a dynamic game with endogenous group capacity. The game has three key features that are essential to the mowing-the-grass metaphor. First, when the terrorist group invests resources to increase its capacity, the government observes the resulting investment before it decides to attack. That is, the mower can watch the grass grow. Second, capacity is persistent in the sense that once the group acquires capacity, it persists until the government takes costly action to destroy it. That is, the grass does not mow itself. Third, the interaction has an infinite horizon. That is, the grass at least has the opportunity to regrow.

This stylized model elucidates the dynamic tradeoffs underlying mowing the grass. When costs are large enough such that the actors cannot commit to attack or invest in every period, the government only attacks if it expects the group to not immediately rebuild. Likewise, the group only builds its organizational capabilities if it expects the government to not respond immediately with attacks. Thus, the group's dynamic benefits of acquiring capacity intimately depend on its expectations about the frequency of government attacks, and the government's dynamic benefits of attacking depend on its expectation about how quickly the group rebuilds. As such, multiple equilibria exist. In the deterrence equilibrium, the group never invests in its capacity, and the government always attacks groups with high capacity. In the rampant-terrorism equilibrium, the group always invests and the government never attacks. In the mowing-the-grass equilibrium, the group randomizes its investment decision and the government randomizes its attack decision.

The mowing-the-grass equilibrium, therefore, rationalizes cycles of government attacks and the evolution of terrorist capacity over time. It emerges under relatively stark conditions that do not require punishment strategies, incomplete or imperfect information, revenge preferences, or exogenous stochastic forces, which are other explanations for the patterns. It also produces novel comparative statics. In equilibrium, the government attacks high-capacity groups with probability that makes low-capacity groups indifferent between building capacity and not. When the cost of building capacity increases, the government's equilibrium probability of attacking decreases thereby increasing the dynamic benefits of acquiring capacity and compensating the group for its larger investment costs. Thus, higher investment costs increase the long-term probability of high-capacity terrorism and increase the time between government attacks. For similar reasons, as the government's cost of attacking increases, the terrorist group's equilibrium probability of investing in its capacity decreases, which leads to less high-capacity terrorism and longer times between government attacks.

To quantify these relationships, I calibrate the model using data on Palestinian rockets fired from the Gaza Strip, a motivation behind several IDF operations. A major difficulty is that the relevant actions and group capacity levels are difficult to measure directly. For example, raw levels of terrorism cannot directly measure capacity building because they conflate the terrorists' decision to build and government's decision to destroy capacity, and both decisions may involve the actors mixing. Instead, I assume terrorist capacity imperfectly, albeit positively, correlates with observed levels of violence. As discussed below, such an assumption is consistent with well-studied models in the terrorism literature (e.g., Dragu, 2017; Di Lonardo and Dragu, 2021). With this assumption, the variation in rocket attacks over time identifies how latent capacity transitions between high and low states as in a hidden Markov model, even though the capacity level in any given period is unobserved in the data. As the theoretical model structures how the equilibrium strategies influence these transitions, I back out the endogenous equilibrium quantities from the data.

Specifically, I estimate that the expected time between government attacks is 18.8 months and the long-term probability of high-capacity rocket attacks is 0.93, roughly matching the qualitative account by Rubin (2011) who argues that the effects of government attacks on rocket-firing capacity are short lived. The empirical analysis also indicates that mowing the grass best explains the dynamics of rocket firings among the model's three equilibria. These first two results illustrate that the model and the selected mowing-the-grass equilibrium are useful to understand real-world conflict dynamics. Further assuming the mowing the grass equilibrium generates the data allows me to conduct explicit counterfactual exercises. I find that equilibrium behavior is relatively unresponsive to local changes in the government's incentives. In contrast, a 10% decrease in the costs of acquiring rocket-firing capacity would decrease the long-term probability of high-capacity groups by 1%–3% but would also increase rate of government attacks between 10%–30%. The exercise reveals a peace-making dilemma: small policy changes affecting local incentives in the calibrated model will not substantially decrease violence by both actors.

Finally, I explore the robustness of mowing the grass. Although the baseline model has only two possible capacity levels—which is the simplest setup to explicate how strategic uncertainty emerges—the dynamic still arises when capacity levels are more finely grained. I also show that mowing the grass arises even when attacks eliminate the group with positive probability, capacity depreciates for reasons besides attacks, or the group can conceal its investment decisions. What is key is that interaction unfolds with an infinite horizon.

Besides identifying the dynamic tradeoffs underlying mowing the grass, this paper also contributes to the wider conflict literature. First, I provide a dynamic foundation for empirical work highlighting the importance of strategic uncertainty in the interactions between governments and insurgent groups. Jaeger and Paserman (2008: 1602) analyze the timing of Palestinian violence against Israelis and find that groups act in 'deliberately unpredictable way[s]'. They argue that 'given Israel's intelligence capabilities … it is probably optimal for the Palestinians to randomize' (p. 1602). Sonin and Wright (2020) describe a similar phenomenon in the Afghanistan context. In counterinsurgencies, Lyall (2009: 343) describes how Russian forces in the Chechen war use 'barrages at random intervals and of varying duration on random days without evidence of enemy movement'. While strategic uncertainty in conflict is often explained using imperfect information and the cat-and-mouse incentives in the Colonel Blotto game (e.g., Roberson, 2006; Sonin and Wright, 2020), I demonstrate these conditions are not necessary. Here, the novel ingredient is endogenous insurgent capacity that persists until the government uses decisive action to destroy it.

Second, previous work highlights that electoral competition warps the government's incentives to fight terrorists, resulting in suboptimal counterterrorism provisions (Dragu and Polborn, 2014; Di Lonardo, 2019; Bueno de Mesquita, 2007; Dragu, 2017). Majoritarian electoral institutions encourage the formation of terrorist groups (Aksoy and Carter, 2014), and elections can create political violence cycles (Crisman-Cox, 2018; Berrebi and Klor, 2006; Nanes, 2017; Aksoy, 2018). In contrast, mowing the grass reveals a more benign avenue through which electoral competition influences conflict. If the competition decreases the government's discount factor because the politician currently in power is likely to be removed from office, then the frequency of high-capacity terrorism will decrease and the time between government attacks will increase. When the government's discount factor decreases, it does not fully internalize the dynamic benefits of attacking. So in the mowing-the-grass equilibrium, the terrorists' propensity to build capacity decreases, leading to less capable terrorists and government attacks in the long run.

Third, although there is a rich history of connecting models of terrorism to data, the connection is generally limited to using reduced-form regressions to test theoretical implications (e.g., Aksoy, 2018; Nanes, 2017) or using case studies to trace theoretical mechanisms (e.g., Schram, 2022; Spaniel, 2019; Berrebi and Klor, 2006). In contrast, this article illustrates the benefits of model calibration. Theoretically, it quantifies comparative statics in the version of the model most closely tethered to data. Empirically, the model helps to identify the government's and terrorist group's equilibrium strategies using time-series data on rocket firings even though capacity building and government attacks are not measured explicitly. In doing so, equilibrium mixing probabilities microfound the transition parameters of a hidden Markov process, which are often left as a black box in other work using these models to study terrorism (Raghavan et al., 2013; Blackwell, 2018).

1.1. Related Work

Other theoretical accounts help to explain cycles of government attacks, but their mechanisms require a combination of incomplete information, punishment strategies, revenge dynamics, or asymmetrically patient actors. Padró i Miquel and Yared (2012) analyze a repeated principal-agent model with imperfect monitoring in which government attacks punish the group for poor performance. Attacks cycle in equilibrium because the government cannot not directly observe the group's actions but cannot commit to attack in all future periods. Jacobson and Kaplan (2007) study a finite-horizon model with perfect information where government attacks today *increase* terrorist payoffs tomorrow 'via the recruitment of revenge seeking individuals to terror organizations' (p. 773). Cycles can emerge when the terrorists are asymmetrically impatient and do not fully internalize the dynamic revenge benefits of government attacks.

While the analysis below incorporates some features of these models, I depart from their assumptions in important ways. Like Padró i Miquel and Yared (2012), the government is unable to commit to attack in every period, but unlike their work, attacks can cycle even without incomplete information or history-dependent strategies.[1] The results thus help to explain the dynamics of counterinsurgencies when the government has considerable intelligence gathering capabilities or when actors cannot communicate about or coordinate on intricate punishment strategies.[2] Like Jacobson and Kaplan (2007), the interaction is sequential with perfect information, but unlike their work cycles still emerge in this model even with equally patient actors and in the absence of revenge dynamics. In fact, as either actor becomes more patient, the range of parameters supporting mowing the grass expands.

This article also relates to those studying endogenous insurgent capacity in dynamic environments.[3] Hausken and Zhuang (2011) conceptualize capacity as a fixed set of resources that can grow over time if invested. Thus, terrorists face a tradeoff between using the resources today or investing them for greater resources tomorrow. Gibilisco (2021) models a secessionist group's capacity as a function of previous repression by the government where repression feeds resentment tomorrow, thereby increasing capacity. They find that governments may randomize between repression and the granting of independence, and this randomization is part of a cycle of repression and mobilization. In the context of interstate wars, Schram (2021) shows that falling powers may 'hassle' rising powers by using low-level military actions to slow the rising powers' accumulation of military capacity, thereby solving potential commitment problems arising from power shifts.

2. Model

The model is a dynamic game between a government, G, and a terrorist group, T. The actors compete over a countably infinite number of periods indexed by $t \in \mathbb{N}$. Period t is characterized by an initial level of capacity, $\underline{c}^t \in \{0, 1\}$, where $\underline{c}^t = 0$ represents a period, where the group begins with low capacity and $\underline{c}^t = 1$ a period with high capacity. In the model, capacity represents the persistent resources that the group uses to carry out attacks. For example, a group with high capacity could be one with highly trained followers or a steady access to weapons whereas a low-capacity group has poorly trained followers or intermittent access to weapons.

Given initial capacity $\underline{c}^t$, the interaction within each period t proceeds as follows.

1. Both actors observe the current level of group capacity.
2. The terrorist group chooses whether to invest in its capacity ($a_T^t = 1$) or not ($a_T^t = 0$).[4]
3. The capacity of the group is updated based on the action chosen, where $\overline{c}^t = \max\{\underline{c}^t, a_T^t\}$ is the group's *interim capacity* after the investment decision.

4. The government observes $\overline{c}^t$ and decides whether to attack the group ($a_G^t = 1$) or not ($a_G^t = 0$).
5. Final capacity is $(1 - a_G^t)\overline{c}^t$, and payoffs are accrued.

Given interim capacity $\overline{c}^t$ and an action profile $a^t = (a_T^t, a_G^t)$, the group's per-period utility is

$$u_T(a^t, \overline{c}^t) = (1 - a_G^t)\overline{c}^t - a_T^t \kappa_T. \tag{1}$$

In equation (1), $(1 - a_G^t)\overline{c}^t$ corresponds to the group's benefits from the final capacity level in period t, the value of which is normalized to one. These benefits capture the relative likelihood of successfully committing violence or achieving another goal, which becomes more likely with high capacity.[5] The value $a_T^t \kappa_T$ represents the upfront costs terrorists pay when investing in capacity, where the group pays cost $\kappa_T > 0$ to build capacity, reflecting the difficulties in building and training their ranks or digging tunnels to smuggle weapons.

The government's utility function takes a similar form:

$$u_G(a^t, \overline{c}^t) = 1 - (1 - a_G^t)\overline{c}^t - a_G^t \kappa_G. \tag{2}$$

In equation (2), $1 - (1 - a_G^t)\overline{c}^t$ represents the government's benefits of ensuring that the group has low capacity, which are normalized to 1. These benefits capture the decreased likelihood that the group successfully commits violence or accomplishes a different goal with low capacity. The value $a_G^t \kappa_G$ represents the costs of attacking, where an attack costs $\kappa_G > 0$.[6]

Between periods t and $t + 1$, group capacity has the following law of motion:

$$\underline{c}^{t+1} = (1 - a_G^t)\overline{c}^t = (1 - a_G^t)\max\{\underline{c}^t, a_T^t\}. \tag{3}$$

Equation (3) says that if the group finished period t with high (low) capacity, then its initial capacity is high (low) in period $t + 1$. That is, the group's capacity is *persistent*: once the group builds its capacity, the investment persists until the government destroys it.[7]

Given a sequence of actions and capacities $\{a^t, \overline{c}^t\}_{t=1}^{\infty}$, actor i's payoffs are the discounted sum of per-period utilities, $\sum_{t=1}^{\infty} \delta_i^{t-1} u_i(a^t, \overline{c}^t)$ where $\delta_i \in (0, 1)$ is the discount factor of $i = G, T$.

Throughout I maintain two substantively motivated restrictions on the parameter space; the strategic interaction is trivial without them.

Assumption 1 Costs are not overwhelming large: $\kappa_i < \frac{1}{1-\delta_i}$ for $i = G, T$.

Assumption 1 says that the group prefers to acquire capacity if it knew its capacity would remain high in all future periods. Likewise the government would attack if it expects the group's capacity to remain low. The next assumption imposes a lower bound on the costs of attacking for the government.

Assumption 2 The government's cost of attacking is consequential: $\kappa_G > 1$.
Assumption 2 says the government does not have trivially small costs so that attacking in every period guarantees a positive per-period payoff. Notice that the

assumption does not impose a similar restriction on the group's cost, which can be arbitrarily close to zero.

I focus on stationary and Markovian behavior. Not only is this standard in dynamic games, but the mowing-the-grass metaphor suggests that the government conditions its behavior only on the current capacity level. If the government uses a stationary and Markovian strategy, then the group's best response shares those properties as well. For actor i, a strategy is a function $\sigma_i : \{0, 1\} \to [0, 1]$, where $\sigma_T(\underline{c})$ is the probability that the group invests with initial capacity $\underline{c}$, and $\sigma_G(\overline{c})$ is the probability that the government attacks given interim capacity $\overline{c}$. A strategy profile is $\sigma = (\sigma_T, \sigma_G)$. It is straightforward to define i's continuation value, $V_i^\sigma(\underline{c})$, and i's expected utilities over actions, $U_T^\sigma(a_T; \underline{c})$ and $U_G^\sigma(a_G; \overline{c})$—see Online Appendix A. I focus on Markov Perfect Equilibria, that is, subgame perfect equilibria in stationary and Markovian strategies, referred to as equilibria hereafter.

Profile σ produces a Markov transition matrix describing the evolution of capacity between periods t and $t+1$:

$$M^\sigma \equiv \begin{matrix} \underline{c}^t = 0 \\ \underline{c}^t = 1 \end{matrix} \begin{pmatrix} \underline{c}^{t+1} = 0 & \underline{c}^{t+1} = 1 \\ 1 - \sigma_T(0)(1 - \sigma_G(1)) & \sigma_T(0)(1 - \sigma_G(1)) \\ \sigma_G(1) & 1 - \sigma_G(1) \end{pmatrix}$$

An invariant distribution $\pi^\sigma = (\pi^\sigma(0), \pi^\sigma(1))$ is a probability distribution over $\underline{c} \in \{0, 1\}$ that describes the long-term probability of being at capacity level given profile σ. Furthermore, π^σ is implicitly defined by the equation: $\pi^\sigma M^\sigma = \pi^\sigma$. For a profile σ such that $\sigma_T(0) + \sigma_G(1) > 0$, the long-term probability of high-capacity terrorism is

$$\pi^\sigma(1) = \frac{\sigma_T(0)(1 - \sigma_G(1))}{\sigma_T(0) + \sigma_G(1) - \sigma_T(0)\sigma_G(1)}.$$

Capacity level $\underline{c}$ is absorbing if $\pi^\sigma(\underline{c}) = 1$.[8]

Before proceeding, several remarks are in order. First, higher capacity increases the group's and decreases the government's per-period payoffs. These effects can be microfounded using other counterterrorism models (Dragu, 2017; Di Lonardo and Dragu, 2021).[9] Instead of adding modeling assumptions to microfound the per-period payoffs, I focus on the persistent aspect of capacity: high-capacity groups are those that have made investments that help them carry out violence in the long run, for example, by training their members or by building tunnels to more easily acquire weapons.[10] In contrast, groups that merely possess resources to carry out missions are not necessarily highly capable in this framework. Once used, the resources may not be easily replenished. In later sections, I warp different aspects of capacity's persistence. In one extension, capacity determines the group's survival rate where high-capacity groups are more likely to survive than low-capacity ones. In another, capacity may depreciate for reasons that are exogenous to government attacks, for example, tunnels used to acquire weapons cave in.

Second, the government observes interim capacity $\overline{c}^t$. This explicitly corresponds to the mowing-the-grass metaphor where the mower sees the grass growing and captures situations in which governments have strong intelligence capabilities. The latter situation is relevant for the empirical application in regards to the IDF (Jacobson and Kaplan, 2007; Jaeger and Paserman, 2008; Rubin, 2011). Theoretically, the assumption departs from previous work that uses private information to explain cycles of attacks. As mentioned above, the analysis therefore explains counterinsurgency dynamics in a wider array of scenarios. Of course, it could be the case that the group is able to conceal its capacity for a period of time. I explore this in Online Appendix K with unobservable investment decisions. In this environment with incomplete information mowing the grass emerges and the direction of the equilibrium comparative statics do not change.

Third, there are two capacity levels. This is the simplest setup needed for the results, which has two benefits. Not only does it help to explicate why strategic uncertainty can appear, but also it motivates a straightforward calibration exercise to quantify the model's comparative statics. Of course, there could exist more capacity levels. I explore this in an extension with three capacity levels and demonstrate that mowing the grass and strategic uncertainty still emerge in this setting. Notably, the government could wait until the group acquires two levels of capacity before attacking.

Fourth, there is a unique equilibrium in the stage game, and it is in pure strategies. Under Assumption 2, the government never attacks for any interim capacity level $\overline{c}$, and the group invests if and only if $\underline{c} = 0$ and $\kappa_T < 1$. Mowing the grass or strategic uncertainty more generally is not possible in a one-shot interaction. This conclusion unsurprisingly generalizes when the two actors interact for any finite number of periods—see Online Appendix L. Thus, an infinite horizon is a necessary condition for mowing the grass or other nontrivial dynamics to emerge in this perfect-information framework.

3. Equilibria

With high initial capacity, the group's interim capacity is high regardless of its investment decision. Likewise, with low-interim capacity, capacity will remain low at the end of period, regardless of the government's attack decision. Because attacking and investing are costly, the group and the government avoid the actions with high initial capacity and low-interim capacity, respectively.

Lemma 1. In every equilibrium σ, the terrorists do not invest if their capacity is high ($\sigma_T(1) = 0$), the government does not attack if the group's capacity is low ($\sigma_G(0) = 0$), and $0 \leq V_i^\sigma(\underline{c}) \leq \frac{1}{1-\delta_i}$ for all $\underline{c} \in \{0, 1\}$ and all actors $i = G, T$.

The proof of Lemma 1 is straightforward and is omitted. The result implies that, in equilibrium, we only need to characterize the group's investment strategy in low-capacity states, $\sigma_T(0)$, and the government's attack strategy in high-capacity states, $\sigma_G(1)$. The main result demonstrates that strategic uncertainty can be one explanation for the cycles of violence between a terrorist organization and government.

Proposition 1. Three equilibria exist. Along with Lemma 1, they take following form.

1. **Mowing the grass**: The government attacks high-capacity terrorists with probability strictly between zero and one, and low-capacity terrorists invest with

probability strictly between zero and one, specifically,

$$\sigma_G(1) = \frac{1-\kappa_T(1-\delta_T)}{1+\delta_T\kappa_T} \quad \text{and} \quad \sigma_T(0) = \frac{1-\kappa_G(1-\delta_G)}{\kappa_G\delta_G}.$$

No capacity level is absorbing ($0 < \pi^\sigma(1) < 1$).

2. **Deterrence**: The government always attacks high-capacity terrorists ($\sigma_G(1) = 1$), and low-capacity terrorists never invest ($\sigma_T(0) = 0$). Low capacity is absorbing ($\pi^\sigma(0) = 1$).
3. **Rampant terrorism**: The government never attacks high-capacity terrorists ($\sigma_G(1) = 0$), and low-capacity terrorists always invest ($\sigma_T(0) = 1$). High capacity is absorbing ($\pi^\sigma(1) = 1$).

The key to understanding Proposition 1 is that the future benefits from attacking or investing are endogenous and depend on equilibrium expectations. The terrorists' benefits from building capacity depends on the likelihood that the government attacks high-capacity groups, and the government's benefits from attacking depends on how frequently low-capacity terrorists build capacity.

This indeterminacy arises from the repeated interaction and generates both the multiple equilibria and the possibility for nontrivial strategic uncertainty. For the latter, low-capacity groups randomize to make the government indifferent between attacking high-capacity groups or not. Likewise, the government attacks high-capacity groups with probability that makes low-capacity groups indifferent between investing or not. In this sense, the strategic environment is similar to a game of chicken, where one actor wants to stand firm (take its costly action) only if it expects the other opponent not to follow. In this game, however, these incentives are generated via an intertemporal substitution: taking the costly action to build capacity or attack is only beneficial if the group or the government does not expect substantial attacks or investment in the future, respectively. Thus, uncertainty about future behavior sustains the actors' indifference conditions, so mowing the grass, and strategic uncertainty more generally, is not possible with a finite number of periods.

Proposition 1 also demonstrates that the same conditions generating mowing the grass are also responsible for multiple equilibria. The model can therefore explain variation in conflict dynamics across seemingly identical cases via equilibrium selection. This multiplicity thwarts efforts to leverage cross-sectional variation when studying the relationship between security policies or political institutions on observed levels of terrorism. These threats to inference are particularly acute if the same historical factors determine equilibrium selection and the independent variables of interest such as political institutions and economic development.

In equilibrium σ, i's long-term expected utility is

$$\tilde{U}_i(\sigma) = \sum_{\underline{c}} \pi^\sigma(\underline{c}) V_i^\sigma(\underline{c}),$$

and i prefers equilibrium σ to equilibrium σ' if $\tilde{U}_i(\sigma) > \tilde{U}_i(\sigma')$. The next result states that the government and terrorists have intuitive preferences over the three equilibria: mowing the grass is never the least-preferred equilibrium for either actor, which is one reason it might be focal.

Proposition 2. The terrorists prefer rampant terrorism to mowing the grass to deterrence. The government prefers deterrence to mowing the grass to rampant terrorism.

If σ^D and σ^R are the deterrence and rampant-terrorism equilibria, respectively, then $\tilde{U}_T(\sigma^D) = \tilde{U}_G(\sigma^R) = 0$. In contrast, if σ^M is the mowing-the-grass equilibrium, then $\tilde{U}_i(\sigma^M) > 0$ for both actors i. Comparing across the three equilibria, mowing the grass maximizes the product of long-term expected utilities, so it is the Nash bargaining solution, which is another explanation for why actors would coordinate on it.

Finally, other factors may influence mowing-the-grass dynamics, for example, private information, revenge preferences, punishment strategies, and failed attacks or investment decisions. The analysis thus far demonstrates that these features are not necessary to generate cycles of government attacks and evolving group capacity. In isolation they do not necessarily explain the empirical record where terrorist groups and governments behave in deliberately unpredictable ways. In Section 6. and the Online Appendix, I explore the robustness of mowing the grass by considering environments in which capacity may correlate with group survival, more than two capacity levels are feasible, or the government cannot perfectly observe investment decisions. Mowing the grass driven by strategic uncertainty still emerges in these settings, and the comparative statics in the baseline model are robust.

4. Comparative statics

There are several reasons to prioritize mowing the grass for comparative statics although multiple equilibria exist. Empirically, it is the only equilibrium that rationalizes cycles of government attacks and the nontrivial evolution of group capacity. In the deterrence or rampant terrorism equilibrium in contrast, the government attacks at most once along the equilibrium path and there exists an absorbing capacity level. Furthermore, in the model calibration exercise in Section 5, the analysis indicates that mowing the grass is the relevant equilibrium when explaining changes in Palestinian rocket capacity over time. This matches other evidence indicating that Palestinian groups deliberately behave unpredictably (Jaeger and Paserman, 2008, 2009). Theoretically, as described above, the equilibrium can be interpreted as an outcome in Nash bargaining over what equilibrium to play. In addition, behavior in the pure-strategy equilibria does not respond to changes in the underlying parameters. As such, the comparative statics do not reverse direction by selecting a different equilibrium and hold weakly in all equilibria. The next result follows from the mixed strategies in Proposition 1.

Proposition 3. In the mowing-the-grass equilibrium, the following hold.

1. The probability that the government attacks high-capacity terrorists is strictly increasing in the patience of the terrorist group, δ_T, and strictly decreasing in the cost of investment, κ_T.

2. The probability that low-capacity terrorists invest is strictly increasing in the patience of the government, δ_G, and strictly decreasing in the cost of attacking, κ_G.

To see the intuition behind the result, recall that the government is attacking with probability that makes a low-capacity group indifferent between investing and not. Thus, if the upfront cost of investing (κ_T) increases, the government's propensity to attack decreases, thereby increasing the dynamic benefits of investing and maintaining the group's indifference. Likewise, if the terrorist group becomes more patient (δ_T), then it better internalizes the dynamic benefits of building capacity, which means the government's propensity to attack increases, maintaining the terrorists' indifference between acquiring capacity and not. Similar mechanisms underlie the comparative statics regarding the terrorists' propensity to acquire capacity.[11] For example, if the government's upfront cost of attacking (κ_G) increases, then the low-capacity group's probability of investing decreases, which increases the government's long-term benefits of attacking thereby maintaining its indifference condition.

Proposition 3 has implications for the long-run probability of high-capacity terrorism and the timing of government attacks. Suppose the government attacks in period t, and let the random variable X denote the number periods until the next attack. So the support of X is $\mathbb{N}$ and $x \in \mathbb{N}$ corresponds to the event where the government attacks in period $t + x$. The probability that the government's next attack is in period $t + x$ is

$$\Pr(X = x|\sigma) = \sigma_T(0)\sigma_G(1)\sum_{k=1}^{x}(1 - \sigma_G(1))^{k-1}(1 - \sigma_T(0))^{x-k}.$$

The expected number of periods until the next attack is

$$E[X|\sigma] = \sum_{x=1}^{\infty} x \Pr(X = x|\sigma) = \frac{\sigma_T(0) + \sigma_G(1) - \sigma_T(0)\sigma_G(1)}{\sigma_T(0)\sigma_G(1)},$$

which is strictly decreasing in $\sigma_T(0)$ and $\sigma_G(1)$. In conjunction with Proposition 3, the following result follows from the functional forms of $E[X|\sigma]$ and $\pi^\sigma(1)$.

Proposition 4. In the mowing-the-grass equilibrium, the following hold.

1. The long-term probability of high-capacity terrorism, $\pi^\sigma(1)$, is strictly increasing in δ_G and κ_T and strictly decreasing in δ_T and κ_G.
2. The expected number of periods between government attacks is strictly increasing in the actors' costs, κ_i and strictly decreasing in their patience, δ_i.

Policymakers often want to decrease the likelihood of high-capacity terrorism and increase the time between government attacks. Overall Proposition 4 suggests that they should focus on increasing the government's costs of attacking and decreasing the government's discount factor. Doing so will decrease violence by both actors in the long run. In contrast, changing the groups' incentives will either increase the frequency of high-capacity terrorism or government attacks.[12] Thus, policies that tie the hands of the government—for example, sanctions or diplomatic pressure after major attacks—may be effective at decreasing long-run hostilities.

Furthermore, the result has implications about the effects of elections on the incidence of terrorism because discount factors may correlate with the frequency of government turnover. If the government expects to maintain power for the foreseeable future, it is likely that δ_G is large, but if the government expects to lose upcoming elections, then δ_G may be smaller. Proposition 4 says that the latter condition with frequent turnover would lead to a lower propensity for high-capacity terrorism and lower frequency of government attacks. Thus, competitive elections may have peace-enhancing effects.

Before proceeding, one potential criticism of selecting mowing the grass for comparative statics relates to the best-response stability of mixed strategies in normal-form games. Broadly, mixed equilibria in one-shot normal-form games with strategic complementarities are unstable when myopic players learn to best respond to each other during repeated play (Echenique and Edlin, 2004).[13] It is unclear, however, what these results imply for the model in this article, which is an infinitely repeated game with an endogenous state variable. Because of the infinite horizon, this type of game can be played in full only once, so actors cannot look to past, completed games to conjecture about play in upcoming, future games (as in Echenique and Edlin, 2004, for example). Furthermore, because capacity is an endogenous state variable, after a single period of interaction, actors only observe or learn about behavior in the relevant state. To see this, suppose the interaction in period t begins with high capacity ($\underline{c}^t = 1$). In this period, the government does not observe the behavior of the group with low capacity, so it is unclear how the government should learn about the likelihood of weak groups to invest. A symmetric problem arises for the group when the interaction in period t begins with low capacity ($\underline{c}^t = 0$) and the group does not build.

These complications aside, mowing the grass satisfies other types of stability conditions. It is strongly stable in the sense of Doraszelski and Escobar (2010). In words, an equilibrium is strongly stable if the equilibrium correspondence is locally a continuous function of exogenous parameters. As a consequence, mowing the grass is essential and can thus be approximated by equilibria of nearby games. If the use of mixed strategies is still bothersome, Doraszelski and Escobar (2010) show that, generically, mixed equilibria in discrete dynamic games can be purified with temporary shocks to per-period payoffs.

5. Rocket fire in the Israeli–Palestinian conflict

I now calibrate the model using data from the Israeli–Palestinian conflict. The exercise has three goals. First, I show that the estimated equilibrium strategies closely match stylized patterns identified in the empirical literature on the conflict, for example, the persistence of Hamas rocket capacity and the timing between IDF invasions into the Gaza strip. This gives us additional confidence that the model is a useful tool for understanding real-world conflict dynamics. Second, the analysis indicates that, among the model's three equilibria, mowing the grass best explains the dynamics of rocket firings in the conflict, which demonstrates that the equilibrium is the relevant one for comparative statics. Third, I use the calibrated model to illustrate comparative statics and counterfactual predictions implied by mowing the grass and certain changes in the group's investment cost and the government's attack cost.

The major difficulty when connecting the model to data is that the actions and capacity levels are difficult to measure directly. Although major IDF operations against terrorists groups—especially those involving incursions into the Gaza Strip—are widely documented, government attacks may involve targeted killings or other clandestine operations. In addition, raw levels of terrorism cannot be a direct measure of capacity building because they conflate the terrorists' decision to build capacity and the government's decision to destroy capacity, and both decisions may involve the actors mixing. For example, after observing a period with low violence, should we infer that the terrorists did not build or that they acquired capacity which was then destroyed by the government?

To overcome this, I start with the assumption that terrorist capacity positively (and imperfectly) correlates with observed levels of violence. For example, at the end of period t, final capacity is $(1 - a_G^t) \max\{a_T^t, \underline{c}^t\} = \underline{c}^{t+1}$ in the model, and we observe some level of violence that has a higher expected value when final capacity is large than when final capacity is small. Under suitable conditions, the assumption can be microfounded using a standard model from the literature on terrorism and counterterrorism—see Online Appendix B. With this assumption, I estimate how terrorist capacity—even though it is not directly observed—evolves over time using a hidden Markov model. Comparing the fitted statistical model to the theoretical model's implied law of motion, M^σ, allows me to back out the equilibrium strategies $\sigma_T(0)$ and $\sigma_G(1)$, thereby providing estimates of the associated quantities π^σ and $E[X|\sigma]$ as well. Further assuming that the mowing-the-grass equilibrium generates the data, I then calibrate the costs of attacking and investing for any discount factors using the mixed strategies in Proposition 1.

More specifically, let o^t denote the observed (normalized) level of terrorist violence at the end of period t, where final capacity is $(1 - a_G^t) \max\{a_T^t, \underline{c}^t\} = \underline{c}^{t+1}$. The observation o^t is drawn from one of two distributions:

$$o^t \sim \mathcal{N}\left(m_{\underline{c}^{t+1}}, s^2_{\underline{c}^{t+1}}\right) \text{ for } \underline{c}^{t+1} \in \{0, 1\}, \tag{4}$$

where m_0 corresponds to the mean level of violence when terrorists have low final capacity and m_1 to the mean level when they have high final capacity. The assumption that terrorist capacity positively correlates with observed violence implies $m_1 > m_0$. Similarly, $s_{\underline{c}^{t+1}}$ corresponds to the standard deviation of observed violence when terrorists have capacity $\underline{c}^{t+1}$.[14] The states evolve according to a Markov transition matrix:

$$M_b^a \equiv \begin{matrix} \\ \underline{c}^t = 0 \\ \underline{c}^t = 1 \end{matrix} \begin{pmatrix} \underline{c}^{t+1} = 0 & \underline{c}^{t+1} = 1 \\ 1 - a & a \\ b & 1 - b \end{pmatrix} \tag{5}$$

in which a is the probability of transitioning from state $\underline{c} = 0$ to $\underline{c} = 1$ and b is the probability of transitioning from $\underline{c} = 1$ to $\underline{c} = 0$.

When the state variable (i.e., capacity level) $\underline{c}$ is unobserved, the statistical model in equations (4) and (5) describes a standard hidden Markov process, with parameters

$(a, b, m_0, s_0, m_1, s_1)$ that can estimated (Visser and Speekenbrink, 2010).[15] Furthermore, when the unobserved states evolve according to the model, that is, $M^\sigma = M^a_b$, b pins down the government's equilibrium probability of attacking high-capacity terrorists. Then $\frac{a}{1-b}$ pins down the low-capacity terrorist's probability of building capabilities. Thus, the equilibrium strategies σ can be estimated from observed levels of violence even though the capacity levels $\underline{c}^t$ are unobserved.

As a measure of observed violence o^t, I use the number of rockets fired from the Gaza Strip from Haushofer et al. (2010). Rockets became an increasingly important tactic of violence as Hamas has moved away from suicide attacks since the early 2000s, corresponding to their growth in status among Palestinians. These rockets are often homemade, unguided projectiles that generally have ranges below 20 miles and payloads below 20 kg. Their fatality rate is low although Zucker and Kaplan (2014) accredit this to civil defense measures. Their general purpose is to terrorize and disrupt Israeli life (Rubin, 2011). Getmansky and Zeitzoff (2014) find that these effects are substantial enough to influence Israeli voting behavior, and Elster (2019) demonstrates that actual rocket fire affects voting patterns, not the threat of rocket fire. Although other Palestinian groups, Islamic Jihad in particular, fire rockets, Hamas pioneered their use. Some evidence indicates that the group is responsible for the majority of rockets fired (Amnesty International, 2009). The IDF has often used the destruction of Hamas rocket capacity as a justification for military incursions into Gaza (Rubin, 2011; Taylor, 2021).

The raw data with the daily number of rockets fired are reported in Online Appendix E. Because the number of rockets remains low before 2003 (less than 100 in total), I drop these years from the analysis.[16] I also sum the number of rockets fired by month to account for the time required to rebuild rocket capacity or to launch government attacks.[17] I use the logged values due to skewness in the data. Because other factors may influence the rate of rocket fire—for example, the IDF pulls out from the Gaza Strip in 2006—I detrend the data using a quadratic time trend.[18] All transformations are illustrated in Online Appendix E. To estimate parameters $(a, b, m_0, s_0, m_1, s_1)$, I follow Visser and Speekenbrink (2010) who use maximum likelihood estimation via the EM algorithm.

5.1. Estimates

Table 1 presents the results. The estimates of m_0 and m_1 reveal that the two states are well separated, one where the mean is small and close to zero and another with mean that is large.[19] Substantively, the difference implies that terrorists launch about nine times more rockets in months with high-capacity than in months with low-capacity. In addition, the number of rockets fired has more variance with high-capacity rather than low-capacity. The transition probabilities indicate that the government attacks high-capacity terrorists with probability $\sigma_G(1) = b = 0.05$, but the terrorists acquire capacity with probability $\sigma_T(0) = \frac{a}{1-b} = 0.76$. This implies that the expected time between government attacks is 18.8 months and the long-term probability of high-capacity rocket attacks is 0.93. These numbers roughly match the qualitative account by Rubin (2011) who argues that IDF attacks only have short-lived effects on Palestinian rocket capacity and

Table 1. Estimating equilibrium quantities.

	Model Correspondence	Estimate	Standard Error
a	$\sigma_T(1-\sigma_G(1))$	0.723	0.251
b	$\sigma_G(1)$	0.054	0.036
m_0		0.356	0.173
s_0		0.287	0.115
m_1		2.660	0.110
s_1		0.787	0.086
Number of periods		64	
Log likelihood		−83.922	

Note. Standard errors computed from the outer-product of gradients in parentheses.

reports seven named IDF operations targeting rocket capacity, which are spaced 11 months apart on average.

So far, the analysis is agnostic as to which of the three equilibria generate the data. That is, the estimates of the equilibrium strategies $\sigma_G(1)$ and $\sigma_T(0)$ (and hence π^σ and $E[X|\sigma]$) do not depend on equilibrium selection. Table 1 and the qualitative evidence by Rubin (2011) suggest that mowing the grass is the relevant equilibrium, however. Furthermore, in either the deterrent or rampant-terrorism equilibrium, $\pi^\sigma(1) \in \{0, 1\}$. Using a z-test and the delta method for its standard error, the null hypothesis $\pi^\sigma(1) \geq 0.99$ implies a p-value of < 0.1. The null hypothesis $\pi^\sigma(1) \leq 0.01$ implies a p-value of < 0.01. This indicates that the mowing-the-grass equilibrium best matches the data.

Explicitly assuming that mowing the grass is the equilibrium generating the data implies that the estimates in Table 1 are connected to the attack and investment costs via the relevant mixing probabilities in Proposition 1. To see this, in the mowing-the-grass equilibrium the government's cost of attacking satisfies

$$\kappa_G = (1 - \delta_G(1 - \sigma_T(0)))^{-1}.$$

Using $\sigma_T(0) = \frac{a}{1-b}$ from Table 1, $\kappa_G \in (1, 1.31)$ because $\delta \in (0, 1)$. Similarly, the cost of building capacity satisfies

$$\kappa_T = \frac{1 - \sigma_G(1)}{1 - \delta_T(1 - \sigma_G(1))}.$$

Thus, $\sigma_G(1) = b$ from Table 1 implies $\kappa_T \in (0.95, 17.55)$ as $\delta_T \in (0, 1)$.

5.2. Counterfactuals

Using the calibrated model, I now illustrate the substantive quantities of interest as functions of cost parameters κ_G and κ_T. To do this, I consider two cases where actors are patient ($\delta_i = 0.99$) and impatient ($\delta_i = 0.80$).[20] Given the equilibrium strategies estimated in Table 1, patient actors have costs $\kappa_G = 1.30$ and $\kappa_T = 14.93$, whereas impatient actors have costs $\kappa_G = 1.23$ and $\kappa_T = 3.89$. Notice that the calibrated costs are higher when the actors are patient. Given a fixed equilibrium strategy for actor j (calibrated via Table 1), larger discount factors for actor i correspond to greater dynamic benefits associated with i's costly action ($a_i^t = 1$), implying larger upfront costs (κ_i) are required to maintain i's indifference condition.

Figure 1 presents the counterfactuals corresponding to changes in the government's attack costs. The horizontal axis denotes κ_G, where the shaded vertical lines highlight the costs given the estimated equilibrium strategies from Table 1 and the fixed discount factor $\delta_G \in \{0.8, 0.99\}$. The cost of attacking varies between its theoretical lower bound of one and 2.62, which is a 100% increase from the maximum calibrated value. The three vertical axes represent the substantive quantities of interest: the probability that low-capacity terrorists build (top), the long-term probability of high-capacity terrorism (middle), and the expected time between government attacks (bottom). The dashed horizontal lines demarcate the baseline values from the calibrated equilibrium strategies in Table 1. Table 2 in Online Appendix F reports the corresponding marginal effects.

Notice that trends match the comparative statics in Propositions 3 and 4. When the government has larger upfront costs from attacking, the terrorists acquire capacity less frequently, thereby increasing the dynamic benefits of attacking and maintaining the government's indifference condition. This leads to a lower long-term probability of high-capacity terrorism and therefore less frequent government attacks. More substantially, the analysis indicates that changes to the cost of attacking have small peace-enhancing effects: regardless of the government's patience, a 10% increase in the cost of attacking, κ_G, leads to a 0.1% decrease in the long-term probability of high-capacity terrorism and a 0.1% increase in the time between attacks.

Figure 2 presents the counterfactuals corresponding to the terrorists' investing cost, which is the horizontal axis. The shaded vertical lines denote the costs given the estimated equilibrium strategies from Table 1 and the fixed discount factor $\delta_T \in \{0.8, 0.99\}$. The cost of investing varies between its theoretical lower bound of zero and 26, which is a 50% increase from the maximum calibrated value. The three vertical axes represent the probability that the government attacks high-capacity terrorists (top), the long-term probability of high-capacity terrorism (middle), and the expected time between government attacks (bottom). The dashed horizontal lines demarcate the baseline values in the calibrated model; the marginal effects are in Table 2 in Online Appendix F.

Notice that when the terrorists are impatient ($\delta_T = 0.8$), there is a discontinuity in the predicted values of interest. This discontinuity occurs at $\kappa_T = 5 = \frac{1}{1-\delta_T}$. If the terrorists are impatient and $\kappa_T > 5$, then Assumption 1 does not hold and mowing the grass is not an equilibrium. In this case, the only equilibrium is deterrence, which means the government attacks high-capacity groups with probability one, low-capacity terrorism is an absorbing state, and the time between attacks is not defined.

In Figure 2, if the terrorists' costs of acquiring capacity increases, then the government attacks less frequently, thereby increasing the group's dynamic incentives to build capacity. This leads to a greater long-term probability of high-capacity terrorism and a longer expected time between attacks. Compared to Figure 1, the analysis also reveals that the substantive quantities of interest are more responsive to changes in the terrorist's costs of acquiring capacity, especially with smaller discount factors. With impatient terrorists, a 10% decrease in the cost of investing decreases the long-term probability of

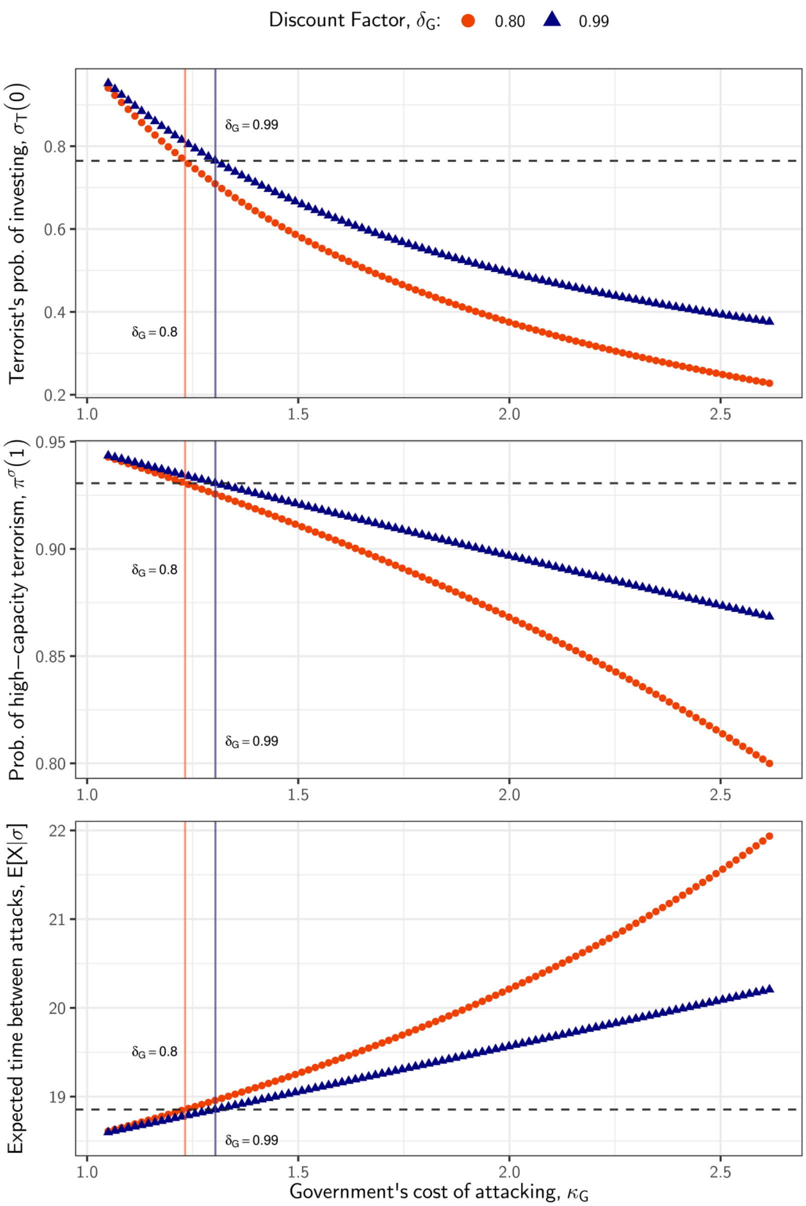

Figure 1. The effects of the government's cost of attacking.
Note. Shaded vertical lines denote the calibrated value of κ_G given patient ($\delta_G = 0.99$) and impatient ($\delta_G = 0.80$) governments using equilibrium estimates from Table 1. The dashed horizontal lines demarcate the baseline quantities in the calibrated model.

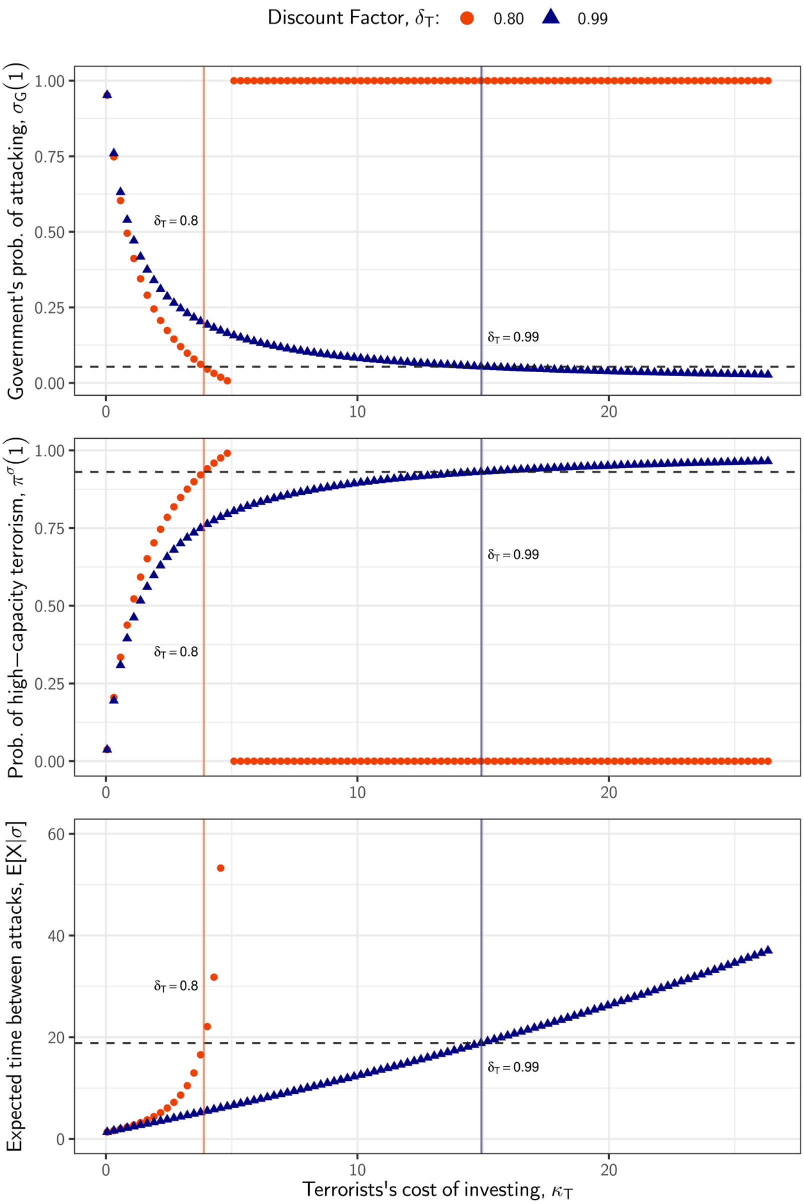

Figure 2. The effects of the terrorists' cost of investing.
Note. Shaded vertical lines denote the calibrated value of κ_T given patient ($\delta_T = 0.99$) and impatient ($\delta_T = 0.80$) terrorists using equilibrium estimates from Table 1. The dashed horizontal lines demarcate the baseline quantities in the calibrated model. When the terrorists are impatient, mowing the grass is not an equilibrium for $\kappa_T > 5 = \frac{1}{1-\delta_T}$, in which case the only equilibrium is deterrence.

high-capacity terrorism by 3% and decreases the expected time between attacks by 31%. With patient terrorists, the effects are diminished to some degree: an identical decrease in the cost of investing decreases the probability of high-capacity terrorism by 1% and the time between attacks by 10%.

Overall, the analysis suggests that policy changes—for example, sanctions after attacks or increasing the cost of capacity through blockades—affecting the actors' relative costs will not substantially decrease both the probability of high-capacity terrorism and the frequency of government attacks. As shown in Proposition 4, only increases in κ_G (or decreases in δ_G) will simultaneously lower the capacity of terrorism and frequency of government attacks. The calibrated model indicates these effects are substantively small, however. In contrast, the equilibrium dynamics are more responsive to changes in the terrorists' incentives, but this has a double-edged sword: altering the costs of acquiring rocket capacity will either increase the long-term probability of terrorism or decrease the frequency of government attacks. Furthermore, if the goal is to increase the cost of acquiring capacity so that deterrence is the only equilibrium, the feasibility of this route depends on the patience of the terrorists. With patient terrorists, even if costs double, mowing the grass is still an equilibrium.

6. Mowing the grass is robust

6.1. Group survival

Groups with low capacity may cease to exist or may never have the opportunity to rebuild. Likewise, the government may attack groups with the explicit goal to eradicate the organization. To capture this, I modify the baseline model. At the end of period t, if capacity is low, $(1 - a_G^t)\bar{c}^t = 0$, then the game continues to period $t + 1$ with probability $q \in (0, 1)$. With probability $1 - q$, the group ceases to exist, in which case it consumes $\chi_T \leq 0$ in all future periods and the government consumes $\chi_G \geq 1$. If capacity is high, that is, $(1 - a_G^t)\bar{c}^t = 1$, then the interaction certainly proceeds to period $t + 1$. Thus, q is the per-period survival rate of low-capacity terrorists, and $q = 1$ covers the baseline model. In addition, $\chi_T \leq 0$ accounts for the possibility that group leaders might want to be in charge of low-capacity groups rather than have the group disband altogether. Similarly, $\chi_G \geq 1$ captures the possibility that the government may receive additional benefits, for example, enhanced electoral prospects or a reallocation of budgetary resources, if it eradicates a terrorist organization.

Proposition 5. Mowing the grass is an equilibrium if and only if $\kappa_G > 1 + \frac{(1-q)\delta_G\chi_G}{1-\delta_G}$. In the equilibrium, the government attacks high-capacity terrorists with probability

$$\sigma_G(1) = 1 + \frac{\kappa_T(1 - q\delta_T)}{-1 + q\delta_T(1 + \delta_T\kappa_T - \chi_T) - \delta_T(\kappa_T - \chi_T)}$$

and low-capacity terrorists invest with probability

$$\sigma_T(0) = \frac{(1 - \delta_G)(1 - (1 - q\delta_G)\kappa_G) + (1 - q)\delta_G\chi_G}{q(1 - \delta_G)\delta_G\kappa_G}.$$

Furthermore, $\sigma_G(1)$ and $\sigma_T(0)$ are decreasing in q.

The requirement that $\kappa_G > 1 + \frac{(1-q)\delta_G\chi_G}{1-\delta_G}$ is stronger than Assumption 2; it comes from the change that the government's dynamic benefits from attacking could be quite large, especially if the per-period survival rate of low-capacity terrorists is small and the benefits of permanently defeating the group is large. Notice that if attacks certainly destroy the group ($q = 0$), then the condition reduces to $\kappa_G > 1 + \frac{\delta_G\chi_G}{1-\delta_G}$. Because $\chi_G \geq 1$, this contradicts Assumption 1. In other words, *mowing the grass is an equilibrium if and only if the per-period survival rate of weak groups q is not too small.*

To see why a larger per-period survival rate decreases the probability that low-capacity terrorists acquire capacity, recall that the group mixes to keep the government indifferent between attacking and not. If the per-period survival of low-capacity groups increases, then the government's dynamic benefits from attacking become smaller. To compensate, low-capacity groups invest less frequently in equilibrium, thereby increasing the dynamic benefits of attacking.

A similar logic explains the comparative static for the government's attack probability. The government is mixing to keep the low-capacity terrorists indifferent between investing and not. Having high capacity comes with two types of benefits for the terrorists: survival and higher per-period payoffs. When q increases, low-capacity terrorists have enhanced survival chances, thereby diminishing the relative survival benefits of high capacity. To maintain indifference in equilibrium, the government attacks less.

6.2. Depreciating capacity

How does capacity's persistence affect mowing the grass. Terrorist groups might lose their high capacity due to other factors besides government attacks. To capture this, I modify the baseline model. At the end of period t if capacity is high, that is, $(1 - a_G^t)\overline{c}^t = 1$, then it remains high in the next period ($\underline{c}^{t+1} = 1$) with probability $p \in (0, 1)$. Capacity depreciates in the next period ($\underline{c}^{t+1} = 0$) with complimentary probability $1 - p$. Thus, p represents the persistence of group capacity, and the baseline model assumed $p = 1$. The next result illustrates the effects of depreciating capacity on mowing-the-grass dynamics.

Proposition 6. Mowing the grass is an equilibrium if and only if $\kappa_i < \frac{1}{1-\delta_i p}$ for $i = G, T$. In the equilibrium, the government attacks high-capacity terrorists with probability

$$\sigma_G(1) = \frac{1 - \kappa_T(1 - p\delta_T)}{1 + p\delta_T\kappa_T}$$

and low-capacity terrorists invest with probability

$$\sigma_T(0) = \frac{1 - \kappa_G(1 - p\delta_G)}{p\delta_G\kappa_G}$$

Furthermore, $\sigma_G(1)$ and $\sigma_T(0)$ are increasing in p.

The persistence of capacity is directly linked to the dynamic benefits of investing and attacking. When persistence increases, the dynamic benefits of acquiring capacity increase, and the government attacks with greater probability to maintain the group's indifference between investing and not. Likewise, with greater persistence, capacity is less likely to disappear in the absence of government attacks, which increases the government's relative benefits from attacking. To ensure that the government is indifferent between attacking and not, the group builds capacity with greater frequency. Notice that $\kappa_i < \frac{1}{1-\delta_i p}$ is stronger than Assumption 1. In addition, if the persistence of capacity goes to zero, this condition boils down to $\kappa_i < 1$, which contradicts Assumption 2. In other words, *mowing the grass is an equilibrium if and only if terrorist capacity is sufficiently persistent.*

6.3. Multiple capacity levels

It could be the case that capacity requires more than one period of investment before the group becomes strong enough to warrant government attacks. That is, groups will need to repeatedly invest before they have high capacity. To capture this, I assume that there are three capacity levels, so $\underline{c}^t \in \{0, 1, 2\}$, representing low, medium and high levels, respectively. In this version, the timing of the interaction is the same as the baseline model, but now capacity evolves as follows:

$$\overline{c}^t = \min\{\underline{c}^t + a_T^t, 2\} \quad \text{and} \quad \underline{c}^{t+1} = (1 - a_G^t)\overline{c}^t.$$

Notice that the group can only increase its capacity by one level in period t. In addition, after the government attacks, capacity is reset to zero, regardless of the level of interim capacity at the time of the attack.

Per-period utilities take the form:

$$u_T(a^t, \overline{c}^t) = (1 - a_G^t)v(\overline{c}^t) - a_T^t \kappa_T$$
$$u_G(a^t, \overline{c}^t) = 1 - (1 - a_G^t)v(\overline{c}^t) - a_G^t \kappa_G.$$

The function $v : \{0, 1, 2\} \to [0, 1]$ relates the capacity levels to terrorism outputs, which takes the form

$$v(\overline{c}^t) = \begin{cases} 1 & \text{if } \overline{c}^t = 2 \\ \mu & \text{if } \overline{c}^t = 1 \\ 0 & \text{if } \overline{c}^t = 0 \end{cases}$$

where $\mu \in (0, 1)$ describes the effectiveness of groups with medium capacity. When μ is close to zero, it essentially takes two periods of investment before the terrorists become high quality. When μ is close to one, the strategic interaction looks similar to the baseline model, where the terrorists acquire substantial capacity after one period of investment.

Now a strategy is a function $\sigma_i : \{0, 1, 2\} \to [0, 1]$, where $\sigma_T(\underline{c})$ is the probability that the group invests given initial capacity $\underline{c}$ and $\sigma_G(\overline{c})$ is the probability that the government attacks given interim capacity $\overline{c}$. As in the baseline model, high-capacity terrorists do not invest and the government does not attack low-capacity terrorists in equilibrium. The next proposition illustrates that mowing the grass can still emerge even when the terrorists invest in more than one period to acquire high capacity.

Proposition 7. If $\kappa_T < \frac{\min\{\mu, 1-\mu\}}{1-\delta_T}$ and $1 + \delta_G(1-\mu) < \kappa_G < \frac{1-\mu\delta_G}{1-\delta_G}$, then there exists an equilibrium in partially mixed strategies with the following properties:

- Low-capacity terrorists always invest ($\sigma_G(0) = 1$) and medium-capacity terrorists invest with probability

$$\sigma_T(1) = \frac{1 - (1-\delta_G)\kappa_G - \delta_G\mu}{\delta_G(\kappa_G - 1)}$$

- The government never attacks medium-capacity terrorists ($\sigma_G(1) = 0$) and attacks high-capacity terrorists with probability

$$\sigma_G(2) = \frac{1 - \mu - (1-\delta_T)\kappa_T}{1 + 2\delta_T\kappa_T}.$$

Furthermore, $\sigma_T(1)$ and $\sigma_G(2)$ are decreasing in μ.

The key difference between the dynamics in Proposition 6 and the baseline result is that the government waits until the group acquires two levels of capacity before attacking. Of course, the government still randomizes when facing high-capacity terrorists, or else the group would have no incentive to increase its capacity from medium to high. For the government to mix at capacity $\bar{c} = 2$, the group needs randomize at capacity $\underline{c} = 1$. Thus, the proposition illustrates that strategic uncertainty can still emerge even when the group requires more than one period to gain enough capacity to warrant an attack.

To see the intuition for the comparative statics, note that, when μ increases, (a) the dynamic benefits of attacking decrease for the government and (b) the gains from high capacity relative to medium capacity decrease for the terrorists ($v(2) - v(1)$). The two forces make each actor's costly action less attractive, so the government and the terrorist group attack and invest less in equilibrium to compensate.

6.4. Government capacity

To this point, I have focused on the group's capacity, but it could be the case that the government's capacity is endogenous. This is particularly relevant when attacking in consecutive periods is not feasible due to dynamic resource constraints. For example, after expending military resources to attack the group, the government may need several periods to replenish those resources to be able to attack again. To see how these constraints may affect mowing the grass, I modify the baseline model as follows. First, let $b^t \in \{0, 1\}$ denote an additional endogenous state variable in the model that represents whether the government has resources to attack ($b^t = 1$) or not ($b^t = 0$) in period t. Second, suppose b^t evolves as follows

$$\Pr(b^{t+1} = 1 \mid b^t, a_G^t) = \begin{cases} 1 & \text{if } b^t = 1 = 1 - a_G^t \\ \beta & \text{otherwise} \end{cases}. \tag{6}$$

Equation (6) says that, if government attacks are feasible in period t ($b^t = 1$) and it does not attack in period t ($a_G^t = 0$), then attacks will be feasible in period $t+1$. If attacks are not feasible in period t or if the government attacks in period t, then attacks will be feasible in period $t+1$ with probability $\beta \in (0, 1)$. Thus, β is the rate at which the government's resources recover after an attack, and the baseline model assumed $\beta = 1$.

In this extension, a state is a pair $(\underline{c}, b) \in \{0, 1\} \times \{0, 1\}$. Strategies are functions $\sigma_i : \{0, 1\} \times \{0, 1\} \to [0, 1]$, where $\sigma_T(\underline{c}, b)$ is T's probability of investing given initial capacity $\underline{c}$ and government resources b and $\sigma_G(\overline{c}, b)$ is G's probability of attacking given interim capacity $\overline{c}$ and resources b. The government faces a binding budget constraint when $b = 0$, so $\sigma_G(\overline{c}, 0) = 0$ for all $\overline{c}$. As above, high-capacity terrorists do not invest ($\sigma_T(1, b) = 0$), and the government does not attack low-capacity terrorists in equilibrium ($\sigma_G(0, 1) = 0$). Thus, an equilibrium needs to pin down three quantities: $\sigma_T(0, 0)$, $\sigma_T(0, 1)$ and $\sigma_G(1, 1)$. The first two are the probabilities that low-capacity groups invest when government attacks are infeasible and feasible, respectively. The last is the government's probability of attacking high-capacity groups. In Online Appendix J, I prove the following result that further reduces the potential mixed-strategy equilibria and the potential equilibria in which capacity cycles.

Lemma 2. In every equilibrium σ, $\sigma_T(0, 1) \in (0, 1)$ implies $\sigma_T(0, 0) = 1$, and $\sigma_T(0, 0) < 1$ implies $\sigma_T(0, 1) = 0$. Moreover, in every equilibrium σ such that neither state $(0, 1)$ nor $(1, 1)$ is absorbing, $\sigma_T(0, 0) = 1$ and $\sigma_T(0, 1) \in (0, 1)$.

Lemma 2 implies that in equilibria in which attacks and capacities cycle (i.e., mowing-the-grass equilibria), low-capacity groups invest with probability strictly between zero and one when attacks are feasible and surely invest when attacks are infeasible. Thus, the next result characterizes all mowing-the-grass equilibria in this extensions, that is, equilibria in which $\sigma_T(0, 1) \in (0, 1)$ and $\sigma_T(0, 0) = 1$.

Proposition 8. Mowing the grass is an equilibrium if and only if $\kappa_G < \frac{1-(1-\beta)\delta_G}{1-\delta_G}$ and $\kappa_T > \frac{(1-\beta)\delta_T}{1-(1-\beta)\delta_T^2}$. In the equilibrium, the government attacks high-capacity terrorists with probability

$$\sigma_G(1, 1) = \frac{(1-(1-\beta)\delta_T)(1-(1-\delta_T)\kappa_T)}{1-2(1-\beta)\delta_T^2\kappa_T + \delta_T(-2(1-\beta)+(2-\beta)\kappa_T)};$$

when attacks are feasible, low-capacity terrorists invest with probability

$$\sigma_T(0, 1) = \frac{\delta_G(\beta+\kappa_G-1)-(\kappa_G-1)}{\delta_G(\beta+\kappa_G-1)};$$

and when attacks are infeasible, low-capacity terrorists surely invest ($\sigma_T(0, 0) = 1$). Furthermore $\sigma_G(1, 1)$ is strictly decreasing in β, while $\sigma_T(0, 1)$ is strictly increasing in β.

Thus, mowing the grass is consistent with the government facing a dynamic budget constraint where it may not be feasible to attack in two consecutive periods. Notice the requirements that $\kappa_G < \frac{1-(1-\beta)\delta_G}{1-\delta_G}$ and $\kappa_T > \frac{(1-\beta)\delta_T}{1-(1-\beta)\delta_T^2}$ are stronger than the baseline model. When β is quite small, the group has large dynamic incentives to invest, so indifference may not be feasible for the group. Because of this, the government has very small dynamic incentives to attack when β is small because the government anticipates the group will quickly rebuild. This also makes mixed strategies more difficult to sustain. In other words, *mowing the grass is an equilibrium if and only if government resources replenish sufficiently quickly after an attack.* By Lemma 2, the equilibrium in Proposition 8 is the only equilibrium consistent with cycles of capacity and attacks.[21] Thus, tight budget constraints (i.e., β close to 0) are not sufficient to explain cycles of government attacks.

7. Conclusion

I study a dynamic model of endogenous group capacity to determine under what conditions insurgent groups acquire capacity in the shadow of government attacks. The dynamic benefits of building capacity for the group depend intimately on its expectation of future government attacks. Conversely, the government's dynamic benefits of attacking are tied to how quickly the group rebuilds capacity. With an infinite-horizon, the two actors may have incentives to randomize their behavior over time, so strategic uncertainty helps to explain cycles of government attacks, fluctuations in terrorist capacity, and mowing the grass. Furthermore, the mechanism does not require incomplete or imperfect information, punishment strategies, revenge preferences, or stochastic shocks.

The analysis also leads to novel substantive implications. First, frequent political turnover may dampen long-term hostilities. In the presence of high turnover, government politicians may not internalize the dynamic benefits of degrading terrorist capacity. Under mowing-the-grass dynamics, decreases in the government's patience or discount factor lead to less high-capacity terrorism and government attacks in the long run. Second, when I calibrate the model to Palestinian rocket attacks, I find that changes in the government's incentives have relatively smaller effects on the frequency of violence than changes in the terrorists' incentives. This illustrates a peacemaking dilemma: changes in the terrorists' cost of acquiring rocket capacity may result in less high-capacity terrorism but it would do so at the expense of increasing the frequency of government attacks.

The analysis also motivates future research in several directions. First, one finding is that strategic uncertainty can explain conflict dynamics over time. These incentives appear in other settings, for example, insurgencies, and one possibility is to fully estimate a model using a case with sufficient data. In light of this, strategic uncertainty may help rationalize observed violence without the introduction of ad hoc action-specific shocks or measurement error. Second, I focus the model on the dynamics of counterterrorism policy and persistent group capacity, but the analysis overlooks more political aspects such as competition between rival anti-government groups or bargaining with the government. Future work should explore how these political aspects of violence interact with endogenous group capacity.

Finally, the paper's motivation and empirical analysis focuses on the cyclical interaction between governments and terrorist groups, but the model is more general and could be applied to other types of asymmetric conflict. The key assumptions are that (i) the anti-government group can acquire capacity that persists absent government intervention, (ii) the interaction is potentially infinite, and (iii) the government cannot commit to use intervention against the group in all interactions. Potential examples include how governments use targeted killings against criminal organizations (Calderón et al., 2015; Castillo, 2021), how nationalists repress secessionist groups (Gibilisco, 2021; Lacina, 2014), or how autocrats use covert tactics to suppress opposition movements (Nalepa and Pop-Eleches, 2022; Dragu and Przeworski, 2019). The empirical section illustrates that the model's equilibrium strategies can be estimated even when the actions and state variable are not directly observed by the analyst, which is particularly important when studying covert government actions. The major requirement is that there is an indirect measure of group capacity. With such a measure, future work can examine the degree to which the model explains empirical dynamics in these other asymmetric conflicts using a similar approach in Section 5.

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Supplemental material

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Notes

1. Debs and Monteiro (2014) and Bas and Coe (2016) demonstrate how incomplete information and endogenous military capacity can lead to bargaining break-down and war. Besides their focus on bargaining with incomplete information, their models also differ from the analysis below because they include exogenous stochastic elements and because successful increases in military capacity (representing the acquisition of nuclear weapons) cannot be undone by fighting.
2. I further discuss informational assumptions in the next section. In Online Appendix K, I relax the perfect information assumption, show that mowing the grass still emerges, and verify that the model's comparative statics are robust.

3. For static environments see Bueno de Mesquita (2005), Di Lonardo and Dragu (2021), Schram (2022), and Spaniel (2019).
4. I use invest in capacity, build capacity, and acquire capacity interchangeably.
5. Even when the group does not expect military victory or government concessions, terrorism and other violence boost recruitment, financial resources, public support, and other ‘proximate goals’ that are essential for the group’s day-to-day operations (Crenshaw, 1981; Acosta, 2014; Polo and González, 2020).
6. The government’s cost of attacking is independent of the group’s interim capacity $\bar{c}^t$. This assumption is inconsequential, however. As shown below, the government never attacks low-capacity groups in equilibrium, so the results do not change if κ_G depends on the capacity. Assumptions 1 and 2 could be restated with respect to the government’s cost of attacking high-capacity groups.
7. Period 1’s capacity $\underline{c}^1$ is exogenous. I also consider an extension in which the group loses capacity probabilistically for reasons besides attacks.
8. As demonstrated below, $\sigma_T(0) + \sigma_G(1) > 0$ holds in every equilibrium.
9. See Online Appendix B for an example.
10. Given the restriction to stationary and Markov strategies, the specific value of the initial state $\underline{c}^1$ is inconsequential to the equilibrium characterization. As such, the analysis below covers the case where initial capacity is low, that is, $\underline{c}^1 = 0$.
11. The comparative statics hold even when the government cannot observe the group’s investment decisions. See Proposition 9 in Online Appendix K.
12. It is possible to interpret the group’s discount factor δ_T as a preference parameter invariant to policy changes. Nonetheless, Castillo and Kronick (2020) and Castillo (2021) argue that targeted killings of group leaders make groups more shortsighted, which would correspond to decreases in δ_T here.
13. Mixed equilibria in one-shot normal-form games can have other microfoundations even when they are not best-response stable. The mixed equilibrium in a game of chicken, for example, is evolutionary stable and asymptotically stable in a large population (Weibull, 1997).
14. Because the level of violence would generally be bounded below (i.e., no violence recorded), I expect $s_1 > s_0$.
15. To clearly see this, let $\tilde{c}^t \equiv (1 - a_G^t) \max\{a_T^t, \underline{c}^t\} = \underline{c}^{t+1}$ denote final capacity in period t. By substitution, $o^t \sim \mathcal{N}(m_{\tilde{c}^t}, s^2_{\tilde{c}^t})$.
16. Rubin (2011) details how rocket firings during this time were relatively uncommon and unimportant to IDF security operations.
17. The calendar month is a standard level of analysis when working with time series data on terrorism (Jaeger and Paserman, 2008, 2009; Jacobson and Kaplan, 2007; Aksoy, 2018)
18. Similar results hold if I detrend using a post-2006 fixed effect.
19. Following conventions, Table 1 reports standard errors, which are computed from a numerical approximation of the information matrix via first-differences and the outer-product of gradients. With a relatively small sample and nonlinear likelihood function, their interpretation is tenuous.
20. A period is a calendar month. For patient actors, a benefit of one in the current period is worth 0.88 after 12 periods/months. For impatient actors, this delayed benefit is worth 0.07.
21. When $\sigma_T(0, 1) \in (0, 1)$, and $\sigma_T(0, 0) = 1$, we must have $\sigma_G(1, 1) \in (0, 1)$ in equilibrium. That is, the government must mix in state $(\underline{c}, b) = (1, 1)$ to make the group indifferent in state (0, 1) because (1, 1) is the only state in which the government can attack with positive probability in equilibrium.

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APPENDIX (online only)

A Technical primitives

Let $\sigma(a, \underline{c})$ denote the probability of observing action profile $a = (a_T, a_G)$ given initial capacity $\underline{c}$ and strategy profile σ. Actor i's continuation value V_i^σ takes the recursive form:

$$V_i^\sigma(\underline{c}) = \sum_{a \in \{0,1\}^2} \sigma(a, \underline{c}) \left[u_i(a, \max\{a_T, \underline{c}\}) + \delta_i V_i^\sigma((1 - a_G) \max\{a_T, \underline{c}\})\right]. \tag{7}$$

Expected utilities over actions take the following form. For the terrorist group,

$$U_T^\sigma(a_T; \underline{c}) = -a_T \kappa_T + \sigma_G(\max\{a_T, \underline{c}\}) \delta_T V_T^\sigma(0) + (1 - \sigma_G(\max\{a_T, \underline{c}\})) \left[\max\{\underline{c}, a_T\} + \delta_T V_T^\sigma(\max\{a_T, \underline{c}\})\right].$$

For the government,

$$U_G^\sigma(a_G; \bar{c}) = 1 - (1 - a_G)\bar{c} - a_G \kappa_G + \delta_G V_G^\sigma((1 - a_G)\bar{c}).$$

Using U_i^σ, it is straightforward to define the equilibrium concept.

Definition 1. *Profile σ is a Markov Perfect Equilibrium if the following conditions hold.*

1. *For all $\underline{c} \in \{0, 1\}$, $\sigma_T(\underline{c}) > 0$ implies $U_T^\sigma(1; \underline{c}) \geq U_T^\sigma(0; \underline{c})$, and $\sigma_T(\underline{c}) < 1$ implies $U_T^\sigma(0; \underline{c}) \geq U_T^\sigma(1; \underline{c})$.*

2. *For all* $\bar{c} \in \{0,1\}$*,* $\sigma_G(\bar{c}) > 0$ *implies* $U_G^\sigma(1;\bar{c}) \geq U_G^\sigma(0;\bar{c})$*, and* $\sigma_G(\bar{c}) < 1$ *implies* $U_G^\sigma(0;\bar{c}) \geq U_G^\sigma(1;\bar{c})$*.*

B Microfoundation of per-period payoffs

In this section, I walk through an example that microfounds the per-period payoffs using a model similar to those in Dragu (2017) and Di Lonardo and Dragu (2021). The example illustrates a model in which (i) excluding costs κ_i, the terrorist group (the government) has higher (lower) per-period benefits with high group capacity and than with low group capacity, and (ii) terrorism or violence is more likely when the group has higher capacity than when the group has lower capacity.

To do this, consider the end of period t. Here, final capacity is $(1-a_G^t)\bar{c}^t = \underline{c}^{t+1}$. Suppose at the end of period t, the government and the group simultaneously choose effort levels $e_i^t \geq 0$ and terrorism or another form of violence (e.g., rocket fire) happens with probability $e_T^t(1-e_G^t)$. In words, the group uses effort to commit terrorism and the government uses effort to stop terrorism. The group receives a benefit of one if and only if terrorism occurs, and it pays effort cost $-\frac{2-\underline{c}^{t+1}}{2}(e_T^t)^2$. Notice that high-capacity groups more easily exert effort than their low-capacity counterparts. The government receives a reduction of negative one if and only if terrorism occurs and pays effort cost $-\frac{1}{2}(e_G^t)^2$. Assume that the effort allocations and the terrorism outcome do not affect the state variable $\underline{c}^t$. That is, the interaction in future periods does not depend on the chosen effort levels in period t or whether or not violence occurred in period t.

With this setup, actor i chooses $e_i^t = \frac{1}{3-\underline{c}^{t+1}}$. Thus, violence occurs with probability

$$e_T^t(1-e_G^t) = \left(\frac{1}{3-\underline{c}^{t+1}}\right)\left(1-\frac{1}{3-\underline{c}^{t+1}}\right) = \underbrace{\frac{2-\underline{c}^{t+1}}{(3-\underline{c}^{t+1})^2}}_{\equiv P_{\underline{c}^{t+1}}}.$$

As such, terrorism and violence are more likely to occur when the group has high final capacity than low final capacity, i.e., $P_1 > P_0$. At the end of the period when actors are deciding effort levels, costs κ_i are sunk. Hence, the group's expected per-period utility (excluding costs κ_T) is

$$P_{\underline{c}^{t+1}} - \frac{2-\underline{c}^{t+1}}{2}\Bigg(\underbrace{\frac{1}{3-\underline{c}^{t+1}}}_{=e_T^t}\Bigg)^2 = \frac{2-\underline{c}^{t+1}}{2(3-\underline{c}^{t+1})^2},$$

which is larger when final capacity is high. The government's expected per-period utility (excluding costs κ_G) is

$$-P_{\underline{c}^{t+1}} - \frac{1}{2}\Bigg(\underbrace{\frac{1}{3-\underline{c}^{t+1}}}_{=e_G^t}\Bigg)^2 = \frac{2\underline{c}^{t+1}-5}{2(3-\underline{c}^{t+1})^2},$$

which is larger when final capacity is low. The payoff difference between high- and low-capacity states can be normalized to one for each actor i relative to its costs κ_i.

C Proof of Proposition 1

I prove the result in two steps. First, I characterize i's best response when $-i$ plays a pure strategy, showing that i's best response is a pure strategy (Lemmas 3 and 4). Second, I characterize mixed strategy equilibria (Lemma 5).

Lemma 3. *1. If low-capacity terrorists always invest ($\sigma_T(0) = 1$), then the government never attacks high-capacity terrorists ($\sigma_G(1) = 0$) in every equilibrium σ.*

2. If low-capacity terrorists never invest ($\sigma_T(0) = 0$), then the government always attacks high-capacity terrorists ($\sigma_G(1) = 1$) in every equilibrium σ.

Proof. To prove (1), consider an equilibrium σ such that $\sigma_T(0) = 1$. We can compute the government's expected utility from attacking when capacity is high:

$$\begin{aligned}
U_G^\sigma(1;1) &= 1 - \kappa_G + \delta_G V_G^\sigma(0) \\
&= 1 - \kappa_G + \delta_G \left[\sigma_G(1) U_G^\sigma(1;1) + (1-\sigma_G(1)) U_G^\sigma(0;1)\right] \\
&= 1 - \kappa_G + \delta_G V_G^\sigma(1) \\
&< 0 + \delta_G V_G^\sigma(1) = U_G^\sigma(0;1),
\end{aligned}$$

where the second equality follows from assumption that $\sigma_T(0) = 1$ and the last inequality follows because $\kappa_G > 1$ by Assumption 2. Because σ is an equilibrium $U_G^\sigma(1;1) < U_G^\sigma(0;1)$ implies that the government never attacks, establishing the desired result.

To prove (2), suppose the contrary. Then there exists an equilibrium σ such that $\sigma_T(0) = 0$ and $\sigma_G(1) < 1$. Because σ is an equilibrium, $\sigma_G(0) = \sigma_T(1) = 0$ by Lemma 1. If the government attacks at interim capacity $\bar{c} = 1$, then its payoff is $U_G^\sigma(1;1) = \frac{1}{1-\delta_G} - \kappa_G$ because capacity will stay low in all future periods when $\sigma_T(0) = 0$. $U_G^\sigma(1;1) > 0$ by Assumption 1. If the government does not attack at interim capacity $\bar{c} = 1$, then its payoff is

$$\begin{aligned}
U_G^\sigma(0;1) &= \delta_G V_G^\sigma(1) \\
&= \delta_G[\sigma_G(1) U_G^\sigma(1;1) + (1-\sigma_G(1)) U_G^\sigma(0;1)] \\
&\leq \delta_G U_G^\sigma(0;1).
\end{aligned}$$

where the inequality follows because $\sigma_G(1) < 1$ implies $U_G^\sigma(1;1) \leq U_G^\sigma(0;1)$ in equilibrium. Because $\delta_G > 0$, $U_G^\sigma(0;1) \leq \delta_G U_G^\sigma(0;1)$ implies $U_G^\sigma(0;1) \leq 0$. But then this means G has a profitable deviation by attacking with probability 1 at $\bar{c} = 1$, which is the desired contradiction. □

Lemma 4. *1. If the government always attacks high-capacity terrorists ($\sigma_G(1) = 1$), then low-capacity terrorists never invest ($\sigma_T(0) = 0$) in every equilibrium σ.*

2. If the government never attacks high-capacity terrorists ($\sigma_G(1) = 0$), then low-capacity terrorists always invest ($\sigma_T(0) = 1$) in every equilibrium σ.

Proof. For (1), consider an equilibrium σ such that $\sigma_G(1) = 1$. Because the government is surely attacking high-capacity terrorists, the terrorist group's expected utility from investing when capacity is low:

$$\begin{aligned} U_T^\sigma(1;0) &= -\kappa_T + \delta_T V_T^\sigma(0) \\ &< 0 + \delta_T V_T^\sigma(0) = U_T^\sigma(0;0). \end{aligned}$$

Thus, T strictly prefers to not invest.

To prove (2), suppose the contrary. Then there exists an equilibrium σ such that $\sigma_G(1) = 0$ and $\sigma_T(0) < 1$. Because σ is an equilibrium, $\sigma_G(0) = \sigma_T(1) = 0$ by Lemma 1. If low-capacity T invests, then the expected payoff is $U_T^\sigma(1;0) = \frac{1}{1-\delta_T} - \kappa_T$ because its capacity will stay large in all future periods when $\sigma_G(1) = 0$. Then $U_T^\sigma(1;0) > 0$ by Assumption 1. Without investment, the expected payoff is

$$\begin{aligned} U_T^\sigma(0;0) &= \delta_T V_T^\sigma(0) \\ &= \delta_T[\sigma_T(0) U_T^\sigma(1;0) + (1-\sigma_T(0)) U_T^\sigma(0;0)] \\ &\leq \delta_T U_T^\sigma(0;0), \end{aligned}$$

where the last inequality follows because $\sigma_T(0) < 1$ implies $U_T^\sigma(0;0) \geq U_T^\sigma(1;0)$ in equilibrium. But then $U_T^\sigma(0;0) \leq 0$ as $\delta_T > 0$, which means T has a profitable deviation to invest with capacity $\underline{c} = 0$. □

Lemma 5. *The unique mixed-strategy equilibrium is the one described in Proposition 1.1.*

Proof. First, consider the group's decision to mix between acquiring capacity and not in state $\underline{c} = 0$. The group's indifference condition takes the following form:

$$U_T^\sigma(0;0) = U_T^\sigma(1;0) \iff \delta_T V_T^\sigma(0) = -\kappa_T + (1-\sigma_G(1))(1+\delta_T V_T^\sigma(1)) + \sigma_G(1)\delta_T V_T^\sigma(0) \quad (8)$$

T's value function in state $\underline{c} = 0$ is

$$\begin{aligned} V_T^\sigma(0) &= \sigma_T(0) U_T^\sigma(1;0) + (1-\sigma_T(0)) U_T^\sigma(0;0) \\ &= U_T^\sigma(0;0) = \delta_T V_T^\sigma(0) = 0, \end{aligned}$$

where second equality follows from T's indifference condition. Substituting $V_T^\sigma(0) = 0$ into Equation 8 implies that the terrorist's indifference condition can be rewritten as

$$0 = -\kappa_T + (1-\sigma_G(1))(1+\delta_T V_T^\sigma(1)). \quad (9)$$

Solving the above indifference condition when $V_T^\sigma(1) = (1-\sigma_G(1))(1+\delta_T V_T^\sigma(1))$ demonstrates that

$$\sigma_G(1) = \frac{1-\kappa_T(1-\delta_T)}{1+\delta_T\kappa_T}.$$

Second, consider the decision of the government to mix with interim capacity $\overline{c} = 1$. G's indifference condition takes the form

$$U_G^\sigma(0;1) = U_G^\sigma(1;1) \iff \delta_G V_G^\sigma(1) = 1 - \kappa_G + \delta_G V_G^\sigma(0). \quad (10)$$

As in the above paragraph, similar arguments show $U_G^\sigma(0;1) = V_G^\sigma(1) = 0$, which means Equation 10 can be written as

$$0 = 1 - \kappa_G + \delta_G V_G^\sigma(0). \tag{11}$$

where the government's value function with low initial capacity is

$$\begin{aligned} V_G^\sigma(0) &= \sigma_T(0)\left[\sigma_G(1)U_G^\sigma(1;1) + (1-\sigma_G(1))U_G^\sigma(0;1)\right] + (1-\sigma_T(0))(1+\delta_G V_G^\sigma(0)) \\ &= \sigma_T(0)U_G^\sigma(0;1) + (1-\sigma_T(0))(1+\delta_G V_G^\sigma(0)) \\ &= (1-\sigma_T(0))(1+\delta_G V_G^\sigma(0)), \end{aligned}$$

where the second equality follows from G's indifference condition. Solving Equation 11 demonstrates that

$$\sigma_T(0) = \frac{1-\kappa_G(1-\delta_G)}{\kappa_G \delta_G}. \qquad \square$$

D Proof of Proposition 2

Let σ^D denote the deterrence equilibrium. Then $\pi^{\sigma^D}(0) = 1$, and $V_T^{\sigma^D}(0) = 0$ and $V_G^{\sigma^D}(0) = \frac{1}{1-\delta_G}$. Let σ^R denote the rampant terrorism equilibrium. Then $\pi^{\sigma^R}(1) = 1$, and $V_T^{\sigma^R}(1) = \frac{1}{1-\delta_T}$ and $V_G^{\sigma^R}(1) = 0$. So $\tilde{U}_T(\sigma^D) = 0 = \tilde{U}_G(\sigma^R)$, $\tilde{U}_T(\sigma^R) = \frac{1}{1-\delta_T}$, and $\tilde{U}_G(\sigma^D) = \frac{1}{1-\delta_G}$.

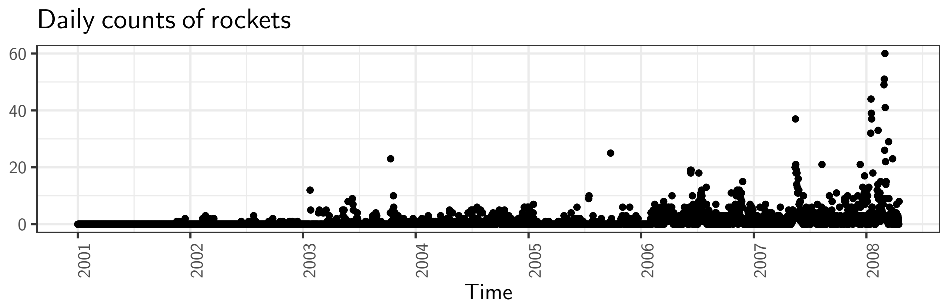
Let σ^M denote the mowing-the-grass equilibrium. In the proof of Proposition 1, we showed that $V_T^{\sigma^M}(0) = V_G^{\sigma^M}(1) = 0$. Equation 11 implies that $V_G^{\sigma^M}(0) = \frac{\kappa_G - 1}{\delta_G}$. Because $\pi^{\sigma^M}(\underline{c}) \in (0,1)$ for $\underline{c} \in \{0,1\}$, $\tilde{U}_G(\sigma^M)$ is a convex combination of 0 and $\frac{\kappa_G-1}{\delta_G}$. By Assumption 2, $\frac{\kappa_G-1}{\delta_G} > 0 = \tilde{U}_G(\sigma^R)$. By Assumption 1, $\frac{\kappa_G-1}{\delta_G} < \frac{1}{1-\delta_G} = \tilde{U}_G(\sigma^D)$, which establishes the government's preference ordering over the three equilibria.

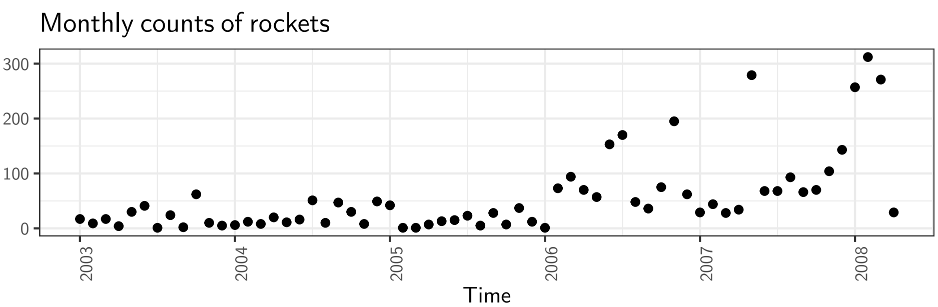
For the terrorists, Equation 9 can be written as

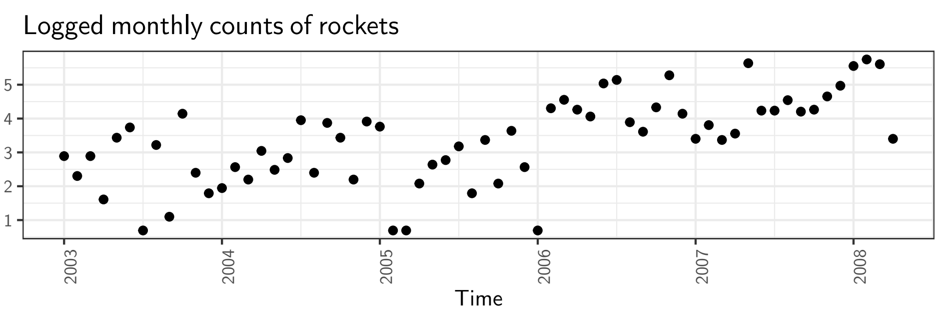
$$\kappa_T = (1-\sigma_G^M(1))(1+\delta_T V_T^{\sigma^M}(1)) = V_T^{\sigma^M}(1).$$

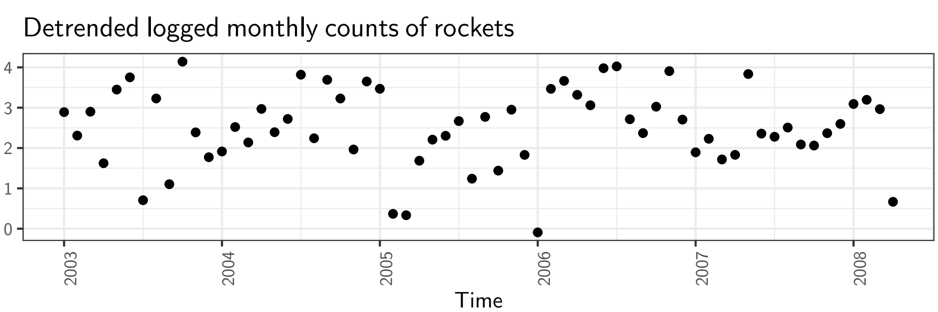
So $V_T^{\sigma^M}(1) = \kappa_T$. Because $\pi^{\sigma^M}(\underline{c}) \in (0,1)$ for $\underline{c} \in \{0,1\}$, $\tilde{U}_T(\sigma^M)$ is a convex combination of 0 and κ_T. By Assumption 1, $\kappa_T \in \left(0, \frac{1}{1-\delta_T}\right)$, which establishes the terrorist group's preference ordering over the three equilibria.

E Data transformation









F Marginal effects in the calibrated model

Table 2: Marginal effects of κ_i.

	δ_{-i}	$\sigma_T(0)$	$\sigma_G(1)$	$\pi^\sigma(1)$	$E[x\|\sigma]$
κ_G	0.80	-0.82		-0.07	1.41
	0.99	-0.59		-0.05	1.02
κ_T	0.80		-0.06	0.07	20.34
	0.99		0.00	0.02	1.38

Notes. The table reports $\frac{\partial f}{\partial \kappa_i}$ given the calibrated strategies in Table 1, where $f \in \{\sigma_T(0), \sigma_G(1), \pi^\sigma(1), E[x|\sigma]\}$ is the equilibrium quantity of interest (from mowing the grass) in the last four columns. The first two columns reference the cost parameter of interest and the two fixed discount factors. The effects correspond to rate of change of the graphs in Figures 1 and 2, where the colored vertical lines meet the dashed horizontal line. Recall that $\sigma_T(0)$ does not depend on κ_T and $\sigma_G(1)$ does not depend on κ_G.

G Proof of Proposition 5 (group survival)

Let $\bar{V}_i$ denote i's continuation value after a terrorist group ceases to exist, so $\bar{V}_T = \frac{\chi_T}{1-\delta_T}$ and $\bar{V}_G = \frac{\chi_G}{1-\delta_G}$. As before $V_i^\sigma(\underline{c})$ denotes i's continuation value when the terrorist group exists with initial capacity $\underline{c} \in \{0,1\}$. Note that after periods where $(1-a_G^t)\bar{c}^t = 1$, the group survives with probability 1, and after all other periods, the group survives with probability q. We characterize the mixed-strategy equilibria where $\sigma_T(0), \sigma_G(1) \in (0,1)$ and $\sigma_T(1) = \sigma_G(0) = 0$.

The group's indifference condition is: $U_T^\sigma(0;0) = U_T^\sigma(1;0)$, which takes the form:

$$\delta_T(qV_T^\sigma(0) + (1-q)\bar{V}_T) = -\kappa_T + \sigma_G(1)\delta_T[qV_T^\sigma(0) + (1-q)\bar{V}_T] + (1-\sigma_G(1))(1+\delta_T V_T^\sigma(1)).$$

We can compute $V_T^\sigma(0)$ as

$$V_T^\sigma(0) = U_T^\sigma(0;0) = \delta_T\left(qV_T^\sigma(0) + (1-q)\bar{V}_T\right) = \frac{(1-q)\delta_T\chi_T}{(1-\delta_T)(1-q\delta_T)}.$$

In addition, $V_T^\sigma(1)$ solves

$$V_T^\sigma(1) = \sigma_G(1)\delta_T\left(qV_T^\sigma(0) + (1-q)\bar{V}_T\right) + (1-\sigma_G(1))(1+\delta_T V_T^\sigma(1)).$$

Solving these three system of equations gives us

$$\sigma_G(1) = 1 + \frac{\kappa_T(1-q\delta_T)}{-1 + q\delta_T(1+\delta_T\kappa_T - \chi_T) - \delta_T(\kappa_T - \chi_T)}.$$

To see that $\sigma_G(1) \in (0,1)$, note that $\sigma_G(1) < 1$ if the denominator in the above fraction is negative, which holds when $\kappa_T > 0$, $\chi_T \leq 0$, and $q, \delta_T \in (0,1]$. In addition , note that

$\sigma_G(1)$ is strictly decreasing in κ_T. Then $\sigma_G(1) > 0$ is equivalent to

$$\begin{aligned}\kappa_T &< \overbrace{\frac{1 - q\delta_T(1-\chi_T) - \delta_T\chi_T}{(1-\delta_T)(1-q\delta_T)}}^{\equiv\bar{\kappa}_T} \\ &\geq \frac{1-q\delta_T}{(1-\delta_T)(1-q\delta_T)} \\ &= \frac{1}{(1-\delta_T)},\end{aligned}$$

where the second inequality follows because $\bar{\kappa}_T$ is decreasing in χ_T and $\chi_T \leq 0$ by assumption. Then Assumption 1 implies $\kappa_T < \frac{1}{(1-\delta_T)}$.

For the government, $U_G^\sigma(0;1) = U_G^\sigma(1;1)$ is equivalent to

$$\delta_G V_G^\sigma(1) = 1 - \kappa_G + \delta_G(qV_G^\sigma(0) + (1-q)\bar{V}_G).$$

Similar arguments from above imply $V_G^\sigma(1) = U_G^\sigma(0;1) = 0$, and $V_G^\sigma(0)$ solves

$$V_G^\sigma(0) = (1-\sigma_T(0))\left[1 + \delta_G(qV_G^\sigma(0) + (1-q)\bar{V}_G)\right].$$

Solving this system of equations gives us

$$\sigma_T(0) = \frac{(1-\delta_G)(1-(1-q\delta_G)\kappa_G) + (1-q)\delta_G\chi_G}{q(1-\delta_G)\delta_G\kappa_G}.$$

To see that $\sigma_T(0) \in (0,1)$, note that $\sigma_T(0)$ is strictly decreasing in κ_G. Then $\sigma_T(0) > 0$ is equivalent to

$$\kappa_G < \overbrace{\frac{1-\delta_G(1-(1-q)\chi_G)}{(1-\delta_G)(1-q\delta_G)}}^{\equiv\bar{\kappa}_G} \geq \frac{1}{1-\delta},$$

where the second inequality follows because $\bar{\kappa}_G$ is increasing in χ_G and $\chi_G \geq 1$. Then Assumption 1 implies $\kappa_G < \frac{1}{1-\delta_G}$. Finally, $\sigma_T(0) < 1$ is equivalent to

$$\kappa_T > 1 + \frac{(1-q)\delta_G\chi_G}{1-\delta_G},$$

which is the necessary and sufficient condition stated in the proposition. Notice that, when $q < 1$, this lower bound on κ_T is larger than Assumption 2.

H Proof of Proposition 6 (depreciating capacity)

We characterize the mixed-strategy equilibria where $\sigma_T(0), \sigma_G(1) \in (0,1)$ and $\sigma_T(1) = \sigma_G(0) = 0$. Recall that the terrorists indifference condition $U_T^\sigma(0;0) = U_T^\sigma(1;0)$ can be written as

$$-\kappa_T + V_T^\sigma(1) = \delta_T V_T^\sigma(0) = 0,$$

where second inequality follows because $U_T^\sigma(0;0) = U_T^\sigma(1;0)$ implies $U_T^\sigma(0;0) = 0$ and $V_T^\sigma(0) = 0$ as in the previous proofs. When the interaction begins in a state with high capacity, capacity is high tomorrow with probability $(1-\sigma_G(1))p$. So the terrorist group's continuation value with high capacity is

$$\begin{aligned} V_T^\sigma(1) &= \sigma_G(1)\delta_T V_T^\sigma(0) + (1-\sigma_G(1))\left(1+\delta_T\left[pV_T^\sigma(1)+(1-p)V_T^\sigma(0)\right]\right) \\ &= (1-\sigma_G(1))\left(1+\delta_T p V_T^\sigma(1)\right). \end{aligned}$$

Solving this system of equations demonstrates that G must be attacking with probability

$$\sigma_G(1) = \frac{1-\kappa_T(1-p\delta_T)}{1+p\delta_T\kappa_T}.$$

Notice $\sigma_G(1)$ strictly decreasing in κ_T. So $\sigma_G(1) < 1$ is equivalent to $\kappa_T > 0$, as assumed. In addition, $\sigma_G(1) > 0$ is equivalent to $\kappa_T < \frac{1}{1-\delta_T p}$, which is half of the necessary and sufficient condition stated in the proposition.

In this version, the government's indifference condition $U_G^\sigma(0;1) = U_G^\sigma(1;1)$ is equivalent to

$$\delta_G\left[pV_G^\sigma(1)+(1-p)V_G^\sigma(0)\right] = 1-\kappa_G+\delta_G V_G^\sigma(0)$$

where the left-hand-side incorporates the possibility that T's capacity depreciates with probability $1-p$ even with no government attacks. Because $U_G^\sigma(0;1) = U_G^\sigma(1;1)$, we can write G's continuation value in the high-capacity state as

$$\begin{aligned} V_G^\sigma(1) &= \sigma_G(1)U_G^\sigma(1;1) + (1-\sigma_G(1))U_G^\sigma(0;1) \\ &= U_G^\sigma(0;1) = \delta_G\left[pV_G^\sigma(1)+(1-p)V_G^\sigma(0)\right] \\ &= \frac{(1-p)V_G^\sigma(0)\delta_G}{1-p\delta_G}. \end{aligned}$$

We can write G's continuation value in the low-capacity state as

$$V_G^\sigma(0) = \sigma_T(0)V_G^\sigma(1) + (1-\sigma_T(0))\left[1+\delta_G V_G^\sigma(0)\right].$$

Solving this system of equations demonstrates that T must acquire capacity with probability

$$\sigma_T(0) = \frac{1-\kappa_G(1-p\delta_G)}{p\delta_G\kappa_G}.$$

Notice that $\sigma_T(0)$ is strictly decreasing in κ_G. So $\sigma_T(0) < 1$ is equivalent to $\kappa_G > 1$ (Assumption 2). Similarly, $\sigma_T(0) > 0$ is equivalent to $\kappa_G < \frac{1}{1-\delta_G p}$, which is the other half of the condition stated in the proposition.

I Proof of Proposition 7 (multiple capacity levels)

Recall that σ takes the form:

$$\sigma_T(2) = \sigma_G(0) = \sigma_G(1) = 0 = 1 - \sigma_T(0)$$
$$\sigma_G(2) = \frac{1 - \mu - (1 - \delta_T)\kappa_T}{1 + 2\delta_T\kappa_T}$$
$$\sigma_T(1) = \frac{1 - (1 - \delta_G)\kappa_G - \delta_G\mu}{\delta_G(\kappa_G - 1)}$$

To see that σ is an equilibrium, we proceed in four steps. First, we characterize the government's equilibrium value functions, V_G^σ, and the terrorists' mixing probability, $\sigma_T(1)$, that makes the government indifferent between attacking and not at high-capacity levels, $\overline{c} = 2$. Second, we make sure that the government has no profitable one-shot deviation at medium-capacity levels, $\overline{c} = 1$. Third, we characterize the terrorist group's equilibrium value functions, V_T^σ, and the government's mixing probability, $\sigma_G(2)$, that makes the terrorist indifferent between investing and not at medium-capacity levels, $\underline{c} = 1$. Fourth, we make sure that the terrorist group has no profitable one-shot deviation at low-capacity levels, $\underline{c} = 1$.

Step 1. G's indifference condition is $U_G^\sigma(1; 2) = U_G^\sigma(0; 2)$:

$$1 - \kappa_G + \delta_G V_G^\sigma(0) = \delta_G V_G^\sigma(2) = 0, \tag{12}$$

where the last inequality follows from now standard arguments as $U_G^\sigma(1; 2) = U_G^\sigma(0; 2)$ implies $V_G^\sigma(2) = 0$. Recall that T builds with probability 1 at $\underline{c} = 0$ and mixes between building and not at $\underline{c} = 1$. So $V_G^\sigma(0) = 1 - \mu + \delta_G V_G^\sigma(1)$. We can write $V_G^\sigma(1)$ as

$$\begin{aligned} V_G^\sigma(1) &= \sigma_T(1)[\sigma_G(2)U_G^\sigma(1; 2) + (1 - \sigma_G(2))U_G^\sigma(0; 2)] + (1 - \sigma_T(1))[1 - \mu + \delta_G V_G^\sigma(1)] \\ &= \sigma_T(1)U_G^\sigma(0; 2) + (1 - \sigma_T(1))[1 - \mu + \delta_G V_G^\sigma(1)] \\ &= (1 - \sigma_T(1))[1 - \mu + \delta_G V_G^\sigma(1)]. \end{aligned}$$

The first inequality follows because $U_G^\sigma(1; 2) = U_G^\sigma(0; 2)$ by assumption. The second follows because $U_G^\sigma(0; 2) = \delta_G V_G^\sigma(2) = 0$. Solving Equation 12 shows that

$$\sigma_T(1) = \frac{1 - (1 - \delta_G)\kappa_G - \delta_G\mu}{\delta_G(\kappa_G - 1)},$$

where $V_G^\sigma(0) = \frac{\kappa_G - 1}{\delta_G}$ and $V_G^\sigma(1) = \frac{\kappa_G - 1 - \delta_G(1-\mu)}{\delta_G^2}$. To see that $\sigma_T(1) > 0$, note that it is strictly decreasing in κ_G. As such,

$$\sigma_T(1) > 0 \iff \kappa_G < \frac{1 - \delta_G\mu}{1 - \delta_G},$$

which is assumed. Likewise,

$$\sigma_T(1) < 1 \iff 1 + \delta_G(1 - \mu) < \kappa_G.$$

Step 2. To ensure G has no profitable deviation at medium capacity level $\overline{c} = 1$, G's payoff from attacking is

$$U_G^\sigma(1;1) = 1 - \kappa_G + \delta_G V_G^\sigma(0) = 1 - \kappa_G + \delta_G \frac{\kappa_G - 1}{\delta_G} = 0.$$

Thus, we need to show

$$V_G^\sigma(1) = \frac{\kappa_G - 1 - \delta_G(1-\mu)}{\delta_G^2} > 0$$

which holds when $\kappa_G > 1 + \delta_G(1-\mu)$, as assumed.

Step 3. T's indifference condition if $U_T^\sigma(0;1) = U_T^\sigma(1;1)$:

$$\mu + \delta_T V_G^\sigma(1) = -\kappa_T + \sigma_G(2)[0 + \delta_T V_T^\sigma(0)] + (1 - \sigma_G(2))[1 + \delta_T V_T^\sigma(2)]. \tag{13}$$

Because $U_T^\sigma(0;1) = U_T^\sigma(1;1)$, we can compute

$$V_T^\sigma(1) = U_T^\sigma(0;1) = \mu + \delta_T V_T^\sigma(1) = \frac{\mu}{1-\delta_T}.$$

In addition, because $\sigma_G(1) = 0 = 1 - \sigma_T(0)$, $V_T^\sigma(0) = \mu - \kappa_T + \delta_T V_T^\sigma(1)$. Finally, $V_T^\sigma(2)$ can be computed as

$$V_T^\sigma(2) = \sigma_G(2)\delta_T V_T^\sigma(0) + (1 - \sigma_G(2))[1 + \delta_T V_T^\sigma(2)].$$

Solving Equation 13 demonstrates that

$$\sigma_G(2) = \frac{1 - (1-\delta_T)\kappa_T - \mu}{1 + 2\delta_T\kappa_T},$$

where $V_T^\sigma(2) = \kappa_T + V_T^\sigma(1)$. To see that $\sigma_G(2) \in (0,1)$, note that $\sigma_G(2)$ is strictly decreasing in κ_T. So $\sigma_G(2) > 0$ if and only if $\kappa_T < \frac{1-\mu}{1-\delta_T}$. Likewise $\sigma_G(2) < 1$ if and only if

$$\kappa_T > -\frac{\mu}{1+\delta}$$

which holds because the fraction above is negative.

Step 4. To see that T wants to acquire capacity at $\underline{c} = 0$ note that

$$\begin{aligned}
U_T^\sigma(0;0) &= \delta_T V_T^\sigma(0) \\
&= \delta_T(\mu - \kappa_T + \delta_T V_T^\sigma(1)) \\
&= \delta_T\left(\mu - \kappa_T + \delta_T \frac{\mu}{1-\delta_T}\right) \\
&= \delta_T(-\kappa_T + V_T^\sigma(1)).
\end{aligned}$$

Because $U_T^\sigma(1;0) = -\kappa_T + V_T^\sigma(1)$, it suffices that

$$V_T^\sigma(1) - \kappa_T > 0 \iff \kappa_T < \frac{\mu}{1-\delta_T},$$

as assumed.

J Proof of Lemma 2 and Proposition 8 (government capacity)

J.1 Proof of Lemma 2

Let $V_i^\sigma(\underline{c}, b)$ denote i's continuation when the interaction starts in state $(\underline{c}, b)$. Recall that $\sigma_G(\overline{c}, 0) = 0 = \sigma_G(0, b)$, for all $\overline{c}, b \in \{0, 1\}$. With this in mind, we write T's continuation values as a system of 4 equations:

$$V_T^\sigma(1,0) = 1 + \delta_T(\beta V_T^\sigma(1,1) + (1-\beta)V_T^\sigma(1,0)) \tag{14}$$
$$V_T^\sigma(1,1) = \sigma_G(1,1)\delta_T(\beta V_T^\sigma(0,1) + (1-\beta)V_T^\sigma(0,0)) + (1-\sigma_G(1,1))(1+\delta_T V_T^\sigma(1,1)) \tag{15}$$
$$V_T^\sigma(0,0) = \sigma_T(0,0)(-\kappa_T + V_T^\sigma(1,0)) + (1-\sigma_T(0,0))\delta_T(\beta V_T^\sigma(0,1) + (1-\beta)V_T^\sigma(0,0)) \tag{16}$$
$$V_T^\sigma(0,1) = \sigma_T(0,1)(-\kappa_T + V_T^\sigma(1,1)) + (1-\sigma_T(0,1))\delta_T V_T^\sigma(0,1) \tag{17}$$

With these continuation values in hand, we prove the lemma in three steps.

Step 1: Show that $\sigma_T(0,1) \in (0,1)$ implies $\sigma_T(0,0) = 1$ in equilibrium σ. Suppose $\sigma_T(0,1) \in (0,1)$ in some equilibrium σ. Then T is indifferent between investing and not given $\underline{c} = 0$ and $b = 1$, i.e., $-\kappa_T + V_T^\sigma(1,1) = \delta_T V_T^\sigma(0,1)$. As in the above proofs, $U_T^\sigma(0; \underline{c} = 0, b = 1) = U_T^\sigma(1; \underline{c} = 0, b = 1)$ and Equation 17 imply that $V_T^\sigma(0,1) = 0$, which means $V_T^\sigma(1,1) = \kappa_T$. Substituting $V_T^\sigma(0,1) = 0$ and $V_T^\sigma(1,1) = \kappa_T$ into Equations 14 and 16 gives us

$$V_T^\sigma(1,0) = 1 + \delta_T(\beta\kappa_T + (1-\beta)V_T^\sigma(1,0)) \tag{14$'$}$$
$$V_T^\sigma(0,0) = \sigma_T(0,0)(-\kappa_T + V_T^\sigma(1,0)) + (1-\sigma_T(0,0))\delta_T((1-\beta)V_T^\sigma(0,0)) \tag{16$'$}$$

Solving Equations 14′ and 16′ shows that

$$V_T^\sigma(1,0) = \frac{1 + \delta_T\beta\kappa_T}{1-(1-\beta)\delta_T} \quad \text{and} \quad V_T^\sigma(0,0) = \frac{(1-(1-\delta_T)\sigma_T(0,0)\kappa_T)}{(1-(1-\beta)\delta_T)(1-(1-\beta)(1-\sigma_T(0,0))\delta_T)}.$$

Now consider the state $(\underline{c}, b) = (0, 0)$. Using the value functions above, we can compute the difference

$$\begin{aligned} U_T^\sigma(1; \underline{c} = 0, b = 0) - U_T^\sigma(0; \underline{c} = 0, b = 0) &= -\kappa_T + V_T^\sigma(1,0) - \delta_T\left(\beta V_T^\sigma(0,1) + (1-\beta)V_T^\sigma(0,0)\right) \\ &= -\kappa_T + V_T^\sigma(1,0) - \delta_T(1-\beta)V_T^\sigma(0,0) \\ &= \frac{1-(1-\delta_T)\kappa_T}{1-(1-\beta)(1-\sigma_T(0,0))\delta_T} > 0. \end{aligned}$$

Above, the last inequality follows via Assumption 1. Thus, $U_T^\sigma(1; \underline{c} = 0, b = 0) > U_T^\sigma(0; \underline{c} = 0, b = 0)$ so $\sigma_T(0,0) = 1$, as required.

Step 2: Show that $\sigma_T(0,0) < 1$ implies $\sigma_T(0,1) = 0$ in equilibrium σ. To see this, suppose $\sigma_T(0,0) < 1$ in some equilibrium σ. Either $\sigma_T(0,1) \in \{0,1\}$. If not, then $\sigma_T(0,1) \in (0,1)$, which means Step 1 implies $\sigma_T(0,0) = 1$, a contradiction. If $\sigma_T(0,1) = 0$, then the proof is complete. To show a contradiction, consider $\sigma_T(0,1) = 1$. By Equation 17,

$\sigma_T(0,1) = 1$ implies $V_T^\sigma(0,1) = -\kappa_T + V_T^\sigma(1,1)$. Because $\sigma_T(0,0) < 1$, $V_T^\sigma(0,0) = U_T^\sigma(0; \underline{c} = 0, b = 0)$. Thus, Equation 16 implies $V_T^\sigma(0,0) = \frac{\beta\delta_T V_T^\sigma(0,1)}{1-(1-\beta)\delta_T}$.

Because $\sigma_T(0,0) < 1$, T must weakly prefer not investing to investing in state $(\underline{c}, b) = (0,0)$, that is,

$$-\kappa_T + V_T^\sigma(1,0) \leq \delta_T(\beta V_T^\sigma(0,1) + (1-\beta)V_T^\sigma(0,0)).$$

Substituting $V_T^\sigma(0,1) = -\kappa_T + V_T^\sigma(1,1)$ and $V_T^\sigma(0,0) = \frac{\beta\delta_T V_T^\sigma(0,1)}{1-(1-\beta)\delta_T}$ gives us

$$-\kappa_T + V_T^\sigma(1,0) \leq \delta_T \left(\beta(-\kappa_T + V_T^\sigma(1,1)) + (1-\beta)\frac{\beta\delta_T(-\kappa_T + V_T^\sigma(1,1))}{1-(1-\beta)\delta_T} \right),$$

and isolating $V_T^\sigma(1,0)$ on the right-hand side implies

$$V_T^\sigma(1,0) \leq \frac{(1-\delta_T)\kappa_T + \beta\delta_T V_T^\sigma(1,1)}{1-(1-\beta)\delta_T}.$$

We can also rewrite Equation 14 to show that $V_T(1,0) = \frac{1+\beta\delta_T V_T(1,1)}{1-(1-\beta)\delta_T} \geq \frac{(1-\delta_T)\kappa_T + \beta\delta_T V_T^\sigma(1,1)}{1-(1-\beta)\delta_T}$, where the inequality follows from the displayed equation above. But that means $(1-\delta_T)\kappa_T \geq 1$, which contradicts Assumption 1.

Step 3: Consider an equilibrium in which neither state $(0,1)$ nor $(1,1)$ is absorbing. Because $(0,1)$ is not absorbing, it must be the case that $\sigma_T(0,1) > 0$. The contrapositive of Step 2 implies $\sigma_T(0,0) = 1$. If $\sigma_T(0,1) = 1$, then T is always investing. As in the proof of Proposition 1, $\kappa_G > 1$ implies that G's best response to T always investing is to not attack, i.e., $\sigma_G(1,1) = 0$. But this means $(1,1)$ is absorbing. So we must have $\sigma_T(0,1) \in (0,1)$.

J.2 Proof of Proposition 8

We characterize equilibria of the form $\sigma_T(0,1) \in (0,1)$ and $\sigma_T(0,0) = 1$. As discussed in the main text, we must also have $\sigma_T(1,b) = 0$ for all $b \in \{0,1\}$, because high-capacity groups have no incentives to invest. For the government, we must have $\sigma_G(\underline{c}, 0) = 0$ because attacks are only feasible in states $(\underline{c}, b)$ with $b = 1$. Likewise, the government will only attack groups with high capacity, so $\sigma_G(0,1) = 0$. Finally, because the group is mixing in state $(\underline{c}, b) = (0,1)$, the government will need to mix (to maintain T's indifference condition) in state $(\underline{c}, b) = (1,1)$, which is the only state where it can potentially mix between attacking and not.

T's indifference condition in state $(\underline{c}, b) = (0,1)$ is

$$\underbrace{-\kappa_T + V_T^\sigma(1,1)}_{U_T^\sigma(1;\underline{c}=0,b=1)} = \underbrace{\delta_T V_T^\sigma(0,1)}_{U_T^\sigma(0;\underline{c}=0,b=1)},$$

where

$$V_T^\sigma(1,1) = \sigma_G(1,1)\delta_T \left[\beta V_T^\sigma(0,1) + (1-\beta)V_T^\sigma(0,0)\right] + (1-\sigma_G(1,1))\left[1 + \delta_T V_T^\sigma(1,1)\right].$$

As in the proofs above, T's indifference condition implies that $V_T^\sigma(0,1) = U_T^\sigma(0; \underline{c} = 0, b = 1) = \delta_T V_T^\sigma(0,1)$, so $V_T^\sigma(0,1) = 0$, which means $V_T^\sigma(1,1) = \kappa_T$. In addition, because $\sigma_T(0,0) = 1$, T is surely investing in state $(0,0)$, so $V_T^\sigma(0,0) = -\kappa_T + V_T^\sigma(1,0)$. Finally, we can write

$$V_T^\sigma(1,0) = 1 + \delta_T \left[\beta V_T^\sigma(1,1) + (1-\beta)V_T^\sigma(1,0)\right].$$

Notice, the continuation values can be written as functions of $\sigma_G(1,1)$ and $V_T^\sigma(1,0)$. Solving this system of 2 equations with 2 unknowns—$\sigma_G(1,1)$ and $V_T^\sigma(1,0)$—gives us

$$V_T^\sigma(1,0) = \frac{1+\beta\delta_T\kappa_T}{1-(1-\beta)\delta_T} \qquad \text{and} \qquad \sigma_G(1,1) = \frac{(1-(1-\beta)\delta_T)(1-(1-\delta_T)\kappa_T)}{1+2(1-\beta)\delta_T^2\kappa_T+\delta(-2+2\beta+2\kappa_T-\beta\kappa_T)}.$$

First, given Assumption 1 and that $\beta, \delta_T \in (0,1)$ implies $\sigma_G(1,1) \in (0,1)$ if and only if $\kappa_T > \frac{\delta_T(1-\beta)}{1-(1-\beta)\delta_T^2}$, which is one restriction in Proposition 8. Finally, Step 1 in the proof of Lemma 2 shows that, if T's indifference condition holds, i.e., $U_T^\sigma(0; \underline{c}=0, b=1) = U_T^\sigma(1; \underline{c}=0, b=1)$, then T has a strict preference to build capacity in state $(\underline{c}, b) = (0,0)$. So T has no profitable deviation when $\sigma_G(1,1)$ takes the form above.

G's indifference condition in state $(\underline{c}, b) = (1,1)$ is

$$\underbrace{1-\kappa_G+\delta_G\left[\beta V_G^\sigma(0,1)+(1-\beta)V_G^\sigma(0,0)\right]}_{U_G^\sigma(1;\underline{c}=1,b=1)} = \underbrace{\delta_G V_G^\sigma(1,1)}_{U_G^\sigma(0;\underline{c}=1,b=1)}.$$

$V_G^\sigma(1,1) = U_G^\sigma(0; \underline{c}=1, b=1) = \delta_G V_G^\sigma(1,1)$ implies $V_G^\sigma(1,1) = 0$. The remainder of G's continuation values take the following form.

$$V_G^\sigma(1,0) = \delta_G\left(\beta V_G^\sigma(1,1)+(1-\beta)V_G^\sigma(1,0)\right) = \delta_G(1-\beta)V_G^\sigma(1,0)$$
$$V_G^\sigma(0,0) = V_G^\sigma(1,0)$$
$$V_G^\sigma(0,1) = \sigma_T(0,1)V_G^\sigma(1,1) + (1-\sigma_T(0,1))(1+\delta_G V_G^\sigma(0,1)) = (1-\sigma_G(0,1))(1+\delta_G V_G^\sigma(0,1)).$$

In the expressions for $V_G^\sigma(1,0)$ and $V_G^\sigma(0,1)$, we invoke the fact that $V_G^\sigma(1,1) = 0$. In the expression for $V_G^\sigma(0,0)$, we invoke the fact that $\sigma_T(0,0) = 1$. Solving this system of equations, along with the indifference condition above gives us

$$V_G^\sigma(0,1) = \frac{\kappa_G - 1}{\beta\delta_G} \qquad \text{and} \qquad \sigma_T(0,1) = \frac{\delta_G(\kappa_G-1+\beta)-(\kappa_G-1)}{\delta_G(\kappa_G-1+\beta)},$$

along with $V_G^\sigma(1,0) = 0$. Assumptions 1 and 2, along with the restrictions that $\delta_G, \beta \in (0,1)$, imply that $\sigma_T(0,1) \in (0,1)$ if and only if $\kappa_G < \frac{1-(1-\beta)\delta_G}{1-\delta_G}$.

K Unobservable investments

The baseline model has perfect information for three reasons: to match the mowing-the-grass metaphor, to show that imperfect or incomplete information was not necessary for government attacks to cycle over time, and to capture situations where the government has a considerable intelligence advantage. The last motivation plausibly captures the empirical application to the IDF in the Israeli-Palestinian conflict (Jacobson and Kaplan 2007; Jaeger and Paserman 2008). It could be the case that governments cannot perfectly observe terrorist capacity, however. Even if the government has substantial intelligence gathering capabilities, the group may be able to hide its investment for a period of time. I account for this possibility in this section.

Specifically, I amend the baseline model as follows. Within each period t, both actors observe the initial level of group capacity, $\underline{c}$. Next, the group chooses whether or not to attack, $a_T^t \in \{0,1\}$. This decision and the interim level of capacity, $\overline{c}$, is unobserved by the government. Finally, the government chooses whether or not to attack and payoffs are accrued. Thus, the group can hide newly acquired capacity for one period. Although the government learns the ultimate capacity level in the next period, it makes its attack decision without knowing the interim level. As such the per-period interaction has a similar flavor to the simultaneous (per-period) interaction in Dragu (2017) and Di Lonardo and Dragu (2021).

As above, I maintain Assumptions 1 and 2 throughout, and I focus on Markov Perfect Equilibria. For actor i, a strategy is a function $\sigma_i : \{0,1\} \to [0,1]$, where $\sigma_T(\underline{c})$ is the probability that the terrorist group invests with initial capacity $\underline{c}$, and $\sigma_G(\underline{c})$ is the probability that the government attacks given initial capacity $\underline{c}$. Unlike the baseline model, the government is unable to condition on interim capacity $\overline{c}$, which is unobserved.

This difference means that the government may attack after observing that initial capacity is low $\underline{c} = 0$ in equilibrium. Like the baseline model, the terrorist group does not invest if their capacity is high ($\sigma_T(1) = 0$) in equilibrium, *but* in this extension there may be different types of mixed strategy equilibria as the government may be mixing when $\underline{c} = 0$ or $\underline{c} = 1$.

Lemma 6. *Suppose σ is an equilibrium such that the low-capacity group acquires capacity with probability strictly between zero and one ($\sigma_T(0) \in (0,1)$).*

1. *If the government is mixing after high initial capacity ($\sigma_G(1) \in (0,1)$), then it never attacks when initial capacity is low ($\sigma_G(0) = 0$).*
2. *If the government is mixing after low initial capacity ($\sigma_G(0) \in (0,1)$), then it always attacks when initial capacity is high ($\sigma_G(1) = 1$).*

Proof. To see the first claim, $\sigma_G(1) \in (0,1)$ in equilibrium σ implies that the government is indifferent between attacking $U_G^\sigma(1;1) = U_G^\sigma(0;1)$:

$$1 - \kappa_G + \delta_G V_G^\sigma(0) = 0 + \delta_G V_G^\sigma(1).$$

Notice that the government's indifference condition implies that $V_G^\sigma(1) = U_G^\sigma(0;1) = \delta_G V_G^\sigma(1)$. Thus, $V_G^\sigma(1) = 0$. Substituting this into the expression above gives us

$$1 - \kappa_G + \delta_G V_G^\sigma(0) = 0,$$

which implies $V_G^\sigma(0) > 0$ as $\kappa_G > 1$ by Assumption 2. Now, consider the government's decision on whether or not to attack when initial capacity is low. If the government attacks, then its payoff is $U_G^\sigma(1;0) = U_G^\sigma(1;1) = 1 - \kappa_G + \delta_G V_G^\sigma(0) = 0$. If the government does not attack, then its payoff is

$$U_G^\sigma(0;0) = \sigma_T(0) \underbrace{\left[0 + \delta_G V_G^\sigma(1)\right]}_{=0} + (1 - \sigma_T(0)) \underbrace{\left[1 + \delta_G V_G^\sigma(0)\right]}_{>0}.$$

Because $\sigma_T(0) \in (0,1)$, $U_G^\sigma(0;0) > 0 = U_G^\sigma(1;0)$.

To see the second claim, $\sigma_G(0) \in (0,1)$ in equilibrium σ implies that the government is indifferent between attacking $U_G^\sigma(1;0) = U_G^\sigma(0;0)$:

$$1 - \kappa_G + \delta_G V_G^\sigma(0) = \sigma_T(0)[0 + \delta_G V_G^\sigma(1)] + (1 - \sigma_T(0))[1 + \delta_G V_G^\sigma(0)].$$

To draw a contradiction, suppose $\sigma_G(1) < 1$. Because σ is an equilibrium, $\sigma_G(1) < 1$ implies $U_G^\sigma(0;1) \geq U_G^\sigma(1;1)$ which means

$$V_G^\sigma(1) = U_G^\sigma(0;1) = 0 + \delta_G V_G^\sigma(1) = 0$$

and

$$U_G^\sigma(0;1) = 0 \geq U_G^\sigma(1;1) = 1 - \kappa_G + \delta_G V_G(0).$$

Substituting $V_G^\sigma(1) = 0$ into the government's indifference condition gives us

$$1 - \kappa_G + \delta_G V_G^\sigma(0) = (1 - \sigma_T(0))[1 + \delta_G V_G^\sigma(0)].$$

The left-hand side of the above expression is $U_G^\sigma(1;1)$, which is weakly less than zero. In addition, the right-hand side is strictly greater than zero as $\sigma_T(0) \in (0,1)$ and $V_G^\sigma(0) \geq 0$. This establishes the desired contradiction. □

The idea behind the Lemma is that (assuming the terrorist group is mixing in its decision to acquire capacity) if the government is indifferent between attacking and not when the group has high capacity at the beginning of the period, then it will not attack groups with low capacity at the beginning of the period. Likewise, if the government is indifferent between attacking and not when the group has low capacity at the beginning of the period, it will surely attack when it knows that the group has high capacity at the beginning of the period. As a consequence, we can reduce our search for mixed strategy equilibria to two cases:

1. Mowing the grass with verification: $\sigma_T(0) \in (0,1)$, $\sigma_G(1) \in (0,1)$, and $\sigma_G(0) = 0$
2. Mowing the grass without verification: $\sigma_T(0) \in (0,1)$, $\sigma_G(0) \in (0,1)$, and $\sigma_G(1) = 1$

With verification, the government waits to attack the group until it sees high capacity although the government is attacking with probability strictly between zero and one. Without verification, the government surely attacks the group when it sees high capacity, and it attacks the group after seeing low capacity with probability strictly between zero and one. The next result says that if $\kappa_T > 1$, then mowing the grass with verification is an equilibrium. The comparative statics of this equilibrium match those in the baseline model.

Proposition 9. *Mowing the grass with verification is an equilibrium if and only if $\kappa_T > 1$. The actors mix with the following probabilities:*

$$\sigma_G(1) = \frac{1 - (1 - \delta_T)\kappa_T}{\delta_T \kappa_T} \quad \textit{and} \quad \sigma_T(0) = \frac{1 - (1 - \delta_G)\kappa_G}{\delta_G \kappa_G}.$$

Comparative statics match the direction of those in the baseline model (Proposition 3).

Proof. First, the group must be indifferent between acquiring and not acquiring capacity with low initial capacity:

$$\underbrace{0 + \delta_T V_T^\sigma(0)}_{U_T^\sigma(0;0)} = \underbrace{1 - \kappa_T + \delta_T V_T^\sigma(1)}_{U_T^\sigma(1;0)}. \tag{18}$$

On the right-hand side, the group is assured to get at least one period of benefit form high capacity because G is only attacking with positive probability after seeing $\underline{c} = 1$ and the group's initial investment decision is hidden. As above, T's indifference condition at $\underline{c} = 0$ implies $V_T^\sigma(0) = 0$. In addition, $V_T^\sigma(1) = \sigma_G(1)\delta_T V_T^\sigma(0) + (1 - \sigma_G(1))[1 + \delta_T V_T^\sigma(1)]$. So we have a system of three equations and three unknowns $(\sigma_G(1), V_T^\sigma(0), V_T^\sigma(1))$ implying that:

$$\sigma_G(1) = \frac{1 - (1 - \delta_T)\kappa_T}{\delta_T \kappa_T} \quad \text{and} \quad V_T^\sigma(1) = \frac{\kappa_T - 1}{\delta_T}.$$

Notice $\sigma_G(1) < 1$ and $V_T^\sigma(1) > 0$ if and only if $\kappa_T > 1$. In addition, Assumption 1 guarantees $\sigma_G(1) > 0$.

Second, the government must be indifferent between attacking and not after seeing high initial capacity, $\underline{c} = 1$. This means

$$\underbrace{0 + \delta_G V_G^\sigma(1)}_{U_G^\sigma(0;1)} = \underbrace{1 - \kappa_G + \delta_G V_G^\sigma(0)}_{U_G^\sigma(1;1)} \tag{19}$$

As above, G's indifference condition at $\underline{c} = 1$ implies $V_G^\sigma(1) = 0$. In addition, $V_G^\sigma(0) = \sigma_T(0)\delta_G V_G(1) + (1 - \sigma_T(0))[1 + \delta_G V_G^\sigma(0)]$. As above, we have a system of three equations and three unknowns ($\sigma_T(0)$, $V_G^\sigma(0)$,$V_G^\sigma(1)$) with the solution as

$$\sigma_T(0) = \frac{1 - (1 - \delta_G)\kappa_G}{\delta_G \kappa_G} \quad \text{and} \quad V_G^\sigma(0) = \frac{\kappa_G - 1}{\delta_G}.$$

Third, the comparative statics follow from straightforward differentiation given the functional forms of the mixed strategies derived above. □

Proposition 10. *Mowing the grass without verification is never an equilibrium.*

Proof. For mowing the grass without verification, the government's indifference condition takes the form:

$$\underbrace{\sigma_T(0)[0 + \delta_G V_G^\sigma(1)] + (1 - \sigma_T(0))[1 + \delta_G V_G^\sigma(0)]}_{U_G^\sigma(0;0)} = \underbrace{1 - \kappa_G + \delta_G V_G^\sigma(0)}_{U_G^\sigma(1;0)}. \tag{20}$$

On the left-hand side, the government is uncertain whether or not the group acquires capacity after observing $\underline{c} = 0$, but this occurs with probability $\sigma_T(0)$. Because the government is surely attacking after observing $\underline{c} = 1$, $V_G^\sigma(1) = 1 - \kappa_G + \delta_G V_G^\sigma(0)$. In addition, after observing $\underline{c} = 0$, the government is indifferent between attacking and not. Thus, $V_G^\sigma(0) = U_G^\sigma(1; 0) = 1 - \kappa_G + \delta_G V_G^\sigma(0)$. Solving this system of equations implies that $\sigma_T(0) = \kappa_G$, but this is infeasible because $\kappa_G > 1$. □

What happens when $\kappa_T < 1$? This is a possibility in the baseline analysis under

Assumptions 1 and 2. The next result says $\kappa_T < 1$ implies that rampant terrorism is the unique equilibrium.

Proposition 11. *If $\kappa_T < 1$, then the only equilibrium is a rampant terrorism equilibrium in which (a) the terrorist group acquires capacity when initial capacity is low ($\sigma_T(0) = 1$) and does not acquire capacity otherwise ($\sigma_T(1) = 0$) and (b) the government never attacks at any capacity level ($\sigma_G(\underline{c}) = 0$).*

Proof. By Lemma 1, mixed strategy equilibria take the form of mowing the grass with or without verification. If $\kappa_T < 1$, then neither equilibrium exists by the previous two propositions. Thus, we focus on pure strategy equilibria. As in Lemma 1, the terrorist group will not invest when capacity is high, $\sigma_T(1) = 0$.

We first argue that $\sigma_T(0) = 1$. If not, then $\sigma_T(0) = 0$, which implies $V_T^\sigma(0) = 0$ and $U_T^\sigma(0; 0) = 0$. Suppose T deviates at $\underline{c} = 0$ by investing once and then returning to strategy σ_T, i.e., of never investing. Then T's payoff is

$$1 - \kappa_T + \delta_T \left[\sigma_G(1) \cdot 0 + (1 - \sigma_G(1)) \frac{1}{1 - \delta_T} \right] > 0,$$

where $\sigma_G(1) = 1$ implies $V_T^\sigma(1) = 0$ and $\sigma_G(1) = 0$ implies $V_T^\sigma(1) = \frac{1}{1-\delta_T}$. But this means T has a profitable deviation. Thus, $\sigma_T(0) = 1$.

To see that $\sigma_G(\underline{c}) = 0$, suppose not. If $\sigma_G(0) = 1$, then $V_G^\sigma(0) = U_G^\sigma(1; 0) = \frac{1-\kappa_G}{1-\delta_G} < 0$. But this means G has a profitable deviation in state $\underline{c} = 0$ by choosing not to attack in all future periods. Thus, $\sigma_G(\underline{c}) = 0$. If $\sigma_G(1) = 1$, then the path of play alternates between $\underline{c} = 0$ and $\underline{c} = 1$, where $V_G^\sigma(0) = 0 + \delta_G V_G^\sigma(1)$ and $V_G^\sigma(1) = 1 - \kappa_G + \delta_G V_G^\sigma(0) < 0$. But this also means G has a profitable deviation after initial capacity $\underline{c} = 1$ by never attacking in all future periods. □

L Finite interaction

What happens without an infinite horizon? Suppose the interaction occurs for a finite number of periods $\mathcal{T} \in \mathbb{N}$, and define $\tau_i^* \in \mathbb{N}_0$ as

$$\tau_i^* \equiv \min \left\{ \tau \in \mathbb{N}_0 \mid \kappa_i < \sum_{t=1}^{\tau} \delta_i^{t-1} \right\} - 1.$$

In words, if the number of future periods is less than or equal τ_i^*, then i's costly action cannot not be (strictly) profitable. Note that τ_i^* is well defined by Assumption 1. In addition, Assumption 2 implies that $\tau_G^* \geq 1$, but it is possible that $\tau_T^* = 0$ if $\kappa_T < 1$. By construction, $\kappa_i \geq \sum_{t=1}^{\tau_i^*} \delta_i^{t-1}$. Generically, this will hold with strict inequality.

Assumption 3. *The game is generic, that is,*

$$\tau_i^* > 0 \text{ implies } \kappa_i > \sum_{t=1}^{\tau_i^*} \delta_i^{t-1}$$

for all i.

In this environment, τ_i^* is a measure of weakness. Specifically, if $\tau_T^* < \tau_G^*$, then the terrorists can credibly commit to acquiring capacity with fewer periods left in the interaction than the government needs to credibly commit to attacking. If $\tau_G^* \leq \tau_T^*$, then the roles are reversed. The government can credibly commit to attacking high-capacity terrorists with fewer remaining periods than the group needs for acquiring capacity to be profitable. The next result demonstrates that the actor with the smaller τ_i^* receives a considerable advantage.

Proposition 12. *Assume $\mathcal{T} > \min\{\tau_T^*, \tau_G^*\}$. In a generic game, the following hold in any Subgame Perfect Nash Equilibrium.*

1. *$\tau_T^* < \tau_G^*$ implies that the government never attacks and that the terrorists build in period t if and only if $\underline{c}^t = 0$ and $t \leq \mathcal{T} - \tau_T^*$. Along the equilibrium path, high-capacity terrorism emerges ($\underline{c}^t = 1$ for all $t > 1$).*

2. *$\tau_G^* \leq \tau_T^*$ implies that the terrorists never acquire capacity and that the government attacks in period t if and only if $\overline{c}^t = 1$ and $t \leq \mathcal{T} - \tau_G^*$. Along the equilibrium path, low-capacity terrorism emerges ($\underline{c}^t = 0$ for all $t > 1$).*

The proof is below. One difference between the statements of Proposition 1 and Proposition 12 is that the latter considers Subgame Perfect Nash Equilibria whereas the former considers only Markov Perfect Equilibria. Perhaps unsurprisingly, in the finite version of the model, the subgame perfect equilibrium strategies only depend on the relevant capacity level ($\underline{c}^t$ or $\overline{c}^t$) and the number of future periods. More substantively, Proposition 12 demonstrates how the potential of continued interaction is necessary to generate mowing-the-grass dynamics with endogenous terrorist capacity and complete information. Mowing the grass or other mixed-strategy equilibria more generally do not emerge with a finite number of interactions.

L.1 Proof of Proposition 12

For $t > 1$, a history is $h^t = \{(\underline{c}^1, a^1), \ldots, (\underline{c}^{t-1}, a^{t-1})\} \in H^t$; for $t = 1$, a history is $h^1 = \underline{c}^1$. Recall $\underline{c}^1$ is an exogenous parameter. To describe the transition of the state variable, define the function

$$G(h^t, t) = \begin{cases} (1 - a_G^{t-1}) \max\{\underline{c}^{t-1}, a_T^{t-1}\} & \text{if } t > 1 \\ h^t & \text{if } t = 1 \end{cases}$$

where $\underline{c}^t = G(h^t, t)$ for all $t > 1$.

In this version of the model, a strategy for the terrorist group is $\sigma_T = \{\sigma_T^t\}_{t=1}^{\mathcal{T}}$ where $\sigma_T^t : H^t \to [0,1]$ and $\sigma_T^t(h^t)$ is the probability of T acquiring capacity in period t after history h^t. A strategy for the government is $\sigma_G = \{\sigma_G^t\}_{t=1}^{\mathcal{T}}$ where $\sigma_G^t : \{0,1\} \times H^t \to [0,1]$ and $\sigma_G^t(a_T^t, h^t)$ is the probability of G attacking in period t after history h^t and T's action a_T^t. Let $V_i^\sigma(h^t)$ denote i continuation value after history h^t given strategy profile σ. As in Lemma 1, if σ is an equilibrium, then $V_i^\sigma(h^t) \in [0, \sum_{t'=\mathcal{T}-t+1}^{\mathcal{T}} \delta^{t'-1}]$ for all i, all h^t and all t.

Lemma 7. *In a generic game with subgame perfect Nash equilibrium σ, the following hold in every period t and history $h^t \in H^t$:*

a) $t > \mathcal{T} - \tau_T^*$ *implies* $\sigma_T^t(h^t) = 0$.

b) $t > \mathcal{T} - \tau_G^*$ *implies* $\sigma_G^t(a_T^t, h^t) = 0$ *for every* $a_T^t \in \{0,1\}$.

Proof. We prove (a), and (b) follows from similar arguments. Consider a subgame perfect Nash equilibrium σ. To derive a contradiction, suppose there exists t such that $t > \mathcal{T} - \tau_T^*$ and $\sigma_T^t(h^t) > 0$. The two cases are $t = \mathcal{T}$ and $\mathcal{T} - \tau_T^* < t < \mathcal{T}$. Consider the latter. For any history $h^t \in H^t$, if T acquires capacity, then its payoff is

$$\begin{aligned} -\kappa_T + \sigma_G^t(1,h^t)\left[0 + \delta_T V_T^\sigma(\{(\underline{c}^1, a^1), \ldots, (\underline{c}^{t-1} a^{t-1}), (G(h^t,t),1,1)\})\right] \\ + (1-\sigma_G^t(1,h^t))\left[1 + \delta_T V_T^\sigma(\{(\underline{c}^1, a^1), \ldots, (\underline{c}^{t-1} a^{t-1}), (G(h^t,t),1,0)\})\right] \end{aligned} \tag{21}$$

At most, T receives 1 in all future periods and 0 at least. Let $\Delta = \mathcal{T} - t$ denote the number of periods in the future. So for any h^{t+1},

$$0 \leq V_T^\sigma(h^{t+1}) \leq \sum_{t'=1}^{\Delta} \delta_T^{t'-1}.$$

Thus, the expression in Equation 21 is bounded above by

$$-\kappa_T + 1 + \delta_T\left(\sum_{t'=1}^{\Delta} \delta_T^{t'-1}\right) = -\kappa_T + \sum_{t'=1}^{\Delta+1} \delta_T^{t'-1} < 0, \tag{22}$$

where the last inequality follows because $\Delta = \mathcal{T} - t$ and $t > \mathcal{T} - \tau^*$ imply $\Delta < \tau_T^*$. Thus, $\Delta + 1 \leq \tau_T^*$.

In contrast, if T does not acquire capacity, then its payoff is

$$\begin{aligned} \sigma_G^t(1,h^T)\left[0 + \delta_T V_T^\sigma(\{(\underline{c}^1, a^1), \ldots, (\underline{c}^{t-1} a^{t-1}), (G(h^t),1,1)\})\right] \\ +(1-\sigma_G^t(1,h^T))\left[G(h^t) + \delta_T V_T^\sigma(\{(\underline{c}^1, a^1), \ldots, (\underline{c}^{t-1} a^{t-1}), (G(h^t),1,0)\})\right]. \end{aligned} \tag{23}$$

Recall that for any h^{t+1}, $0 \leq V_T^\sigma(h^{t+1})$ in equilibrium σ. So a lower bound on T's expected utility for not acquiring capacity is 0, which is strictly larger than the upper bound on T's expected utility for acquiring capacity. So σ cannot be an equilibrium because T can profitably deviate to not investing with probability one.

Finally, for the case where $t = \mathcal{T}$—i.e., the last period—the proof follows along identical lines but is simplified because there are no future interactions. □

Lemma 8. *Assume* $\tau_T^* < \tau_G^*$. *In a generic game with subgame perfect Nash equilibrium* σ, *the following hold in every period* t *and history* $h^t \in H^t$ *such that* $t \leq \mathcal{T} - \tau_T^*$:

1. *the government never attacks, i.e.,* $\sigma_G^t(a_T^t, h^t) = 0$ *for every* $a_T^t \in \{0,1\}$.

2. *terrorists acquire capacity if and only if* $\underline{c}^t = 0$, *i.e.,* $\sigma_T^t(h^t) = 1 - G(h^t,t)$.

Proof. The proof is by induction. For the basis step, we prove the result when $t = \mathcal{T} - \tau_T^*$. Fix $h^t \in H^t$. Because $\tau_T^* < \tau_G^*$, $\mathcal{T} - \tau_T^* > \mathcal{T} - \tau_G^*$, so Lemma 7 implies that G will not attack in period $t = \mathcal{T} - \tau_T^*$. Thus (in the basis step), it suffices to show that T's expected utility from acquiring capacity is strictly larger than its expected utility from not acquiring if and only if $G(h^t,t) = 0$.

Lemma 7 implies that G will not attack in period $t = \mathcal{T} - \tau_T^*$ and in all future periods $t' > t$. Lemma 7 also implies T will not acquire capacity in all future periods $t' > t$. Let $\Delta = \mathcal{T} - t$ denote the number of future periods. If T acquires capacity in period t, its payoff is

$$\begin{aligned} 1 - \kappa_T + \delta_T \sum_{t'=1}^{\Delta} \delta_T^{t'-1} &= -\kappa_T + \sum_{t'=1}^{\Delta+1} \delta_T^{t'-1} \\ &= -\kappa_T + \sum_{t'=1}^{\tau_T^*+1} \delta_T^{t'-1} \\ &> 0. \end{aligned}$$

where the second equality follows because $\Delta = \mathcal{T} - t = \tau_T^*$ and the inequality follows by the construction of τ_T^*. If T does not acquire capacity in period t, then its payoff is

$$\begin{aligned} G(h^t, t) + \delta_T G(h^t, t) \sum_{t'=1}^{\Delta} \delta_T^{t'-1} &= G(h^t, t) \sum_{t'=1}^{\Delta+1} \delta_T^{t'-1} \\ &= G(h^t, t) \sum_{t'=1}^{\tau_T^*+1} \delta_T^{t'-1} \end{aligned}$$

If $G(h^t, t) = 0$, the value above is zero, and if $G(h^t, t) = 1$, the value above is strictly larger than $-\kappa_T + \sum_{t'=1}^{\tau_T^*+1} \delta_T^{t'-1}$. Thus, T's expected utility from acquiring capacity in period t is strictly larger than its expected utility from not acquiring capacity if and only if $G(h^t, t) = \underline{c}^t = 0$, which completes the basis step.

For the induction step, consider a period t such that $t < \mathcal{T} - \tau_T^*$, and suppose the claim holds true for all $t' \in \{t+1, \ldots, \mathcal{T} - \tau_T^*\}$. Fix $h^t \in H^t$. We first argue that the government will not attack when interim capacity is $\overline{c}^t$. Notice that G will not attack in any period $t' \in \{t+1, \ldots, \mathcal{T} - \tau_T^*\}$ by assumption. In addition, G will not attack in any period $t' > \mathcal{T} - \tau_T^*$ by Lemma 7. So G will not attack in any future period. In addition, either $\overline{c}^t = 1$ or $\overline{c}^t = 0$ in which case T would acquire capacity in period $t' + 1$. Thus, G is receiving 0 in all future periods, regardless of its choice of action in period t. Hence, G only attacks if $1 - \kappa_G > 0$, but this not true by Assumption 2.

We next argue that the terrorists acquire capacity if and only if $G(h^t, t) = 0$. Notice that G will not attack in period t and in all future periods. So if the terrorists have capacity at the end of period t, it will persist for all future periods. Let $\Delta = \mathcal{T} - t$ denote the number of future periods. This is strictly larger than the number of future periods in the basis step. As such, an identical logic demonstrates that T will acquire capacity if and only if $G(h^t, t) = 0$. □

Lemma 9. *Assume $\tau_G^* \leq \tau_T^*$. In a generic game with subgame perfect Nash equilibrium σ, the following hold in every period t and history $h^t \in H^t$ such that $t \leq \mathcal{T} - \tau_G^*$:*

1. *the government attacks if and only if $\overline{c}^t = 1$, i.e., $\sigma_G^t(a_T^t, h^t) = \max\{a_T^t, G(h^t, t)\}$.*

2. *terrorists never acquire capacity, i.e., $\sigma_T^t(h^t) = 0$.*

Proof. The proof is by induction. For the basis step, consider $t = \mathcal{T} - \tau_G^*$ and $h^t \in H^t$. First, we show that G's expected utility from attacking is strictly larger than its expected utility from not attacking if and only if $\overline{c}^t = 1$. To do this, note that in all future periods $t' > t = \mathcal{T} - \tau_G^*$, the government never attacks and the terrorists never acquire capacity by Lemma 7 and the assumption that $\tau_G^* \leq \tau_T^*$. Let $\Delta = \mathcal{T} - t$ denote the number of future periods. Then G's payoff from attacking is

$$\begin{aligned} 1 - \kappa_G + \delta_G \sum_{t'=1}^{\Delta} \delta_G^{t'-1} &= -\kappa_G + \sum_{t'=1}^{\Delta+1} \delta_G^{t'-1} \\ &= -\kappa_G + \sum_{t'=1}^{\tau_G^*+1} \delta_G^{t'-1} \\ &> 0 \end{aligned}$$

where the second equality follows because $\Delta = \mathcal{T} - t = \tau_G^*$ and the inequality follows by the construction of τ_G^*. G's payoff from not attacking when interim capacity is $\overline{c}^t = \max\{a_T^t, G(h^t, t)\}$ is

$$(1 - \overline{c}^t) \sum_{t'=1}^{\Delta+1} \delta_G^{t'-1} = (1 - \overline{c}^t) \sum_{t'=1}^{\tau_G^*+1} \delta_G^{t'-1}.$$

If $\overline{c}^t = 1$, the value above is zero, and if $\overline{c}^t = 0$, the value above is strictly larger than G's payoff attacking, which is what we wanted to show.

Second, we show that T does not acquire capacity, in period $t = \mathcal{T} - \tau_G^*$. To do this, note that T will certainly not acquire capacity in any future periods $t' > \mathcal{T} - \tau_G^*$. So *if T acquires capacity* in the current period $t = \mathcal{T} - \tau_G^*$, it will be destroyed by an attack from G (as argued in the above paragraph). So T's payoff from acquiring capacity is $-\kappa_T$. If T does not acquire capacity, its payoff is 0.

For the induction step, consider $t < \mathcal{T} - \tau_G^*$ and suppose the claim holds true for all $t' \in \{t+1, \dots, \mathcal{T} - \tau_G^*\}$. Fix $h^t \in H^t$. Notice that T will not acquire capacity in any future periods. Let $\Delta = \mathcal{T} - t$ denote the number of future periods. This is strictly larger than the number of future periods in the basis step. As such, an identical logic demonstrates that G will attack if and only if $\overline{c}^t = 1$. Therefore, T will never acquire capacity as it expects the capacity to be immediately destroyed and never built in the future periods $t' > t$. □