Tug of War: The Heterogeneous Effects of Outbidding between Terrorist Groups *

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Abstract

We introduce a dynamic game of outbidding where two groups use violence to compete for evolving public support in a tug-of-war fashion. We fit the model to the canonical outbidding rivalry between Hamas and Fatah using newly collected data on Palestinian support for these groups. Competition produces heterogeneous effects, and we demonstrate that intergroup competition can discourage violence. Competition from Hamas leads Fatah to use more terrorism than it would in a world where Hamas abstains from terrorism, but competition from Fatah can lead Hamas to attack less than it otherwise would. Likewise, making Hamas more capable or interested in competing increases overall violence, but making Fatah more capable or interested discourages violence on both sides. This discouragement effect of competition on violence emerges through an asymmetric contest, in which we find that Fatah more effectively uses terrorism to boost its support although Hamas has smaller attack costs. Expanding on these results, we demonstrate that outbidding theory is consistent with a positive, negative, or null relationship between measures of violence and incentives to compete.

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1 Introduction

Outbidding is an explanation for terrorism where competing anti-government groups use violence to increase their share of popular support at the expense of their rivals. In this story, terrorism signals resolve or capacity to a population that is uncertain about which group best represents its interests. In turn, popularity and attention are critical for groups' recruitment numbers, financial resources, political influence, and day-to-day operations (Crenshaw 1981; Fortna 2015; Polo and González 2020). It is a unique theory of terrorism because "the enemy is only tangentially related to the strategic interaction," and therefore outbidding "provides a potential explanation for terrorist attacks that continue even when they seem unable to produce any real results" (Kydd and Walter 2006, 77). Because scholars are still debating the degree to which terrorism helps groups achieve their long-term political objectives (e.g., military victories or government concessions), outbidding provides an important explanation for observed variation in terrorism and intrastate violence.[1]

Following Bloom's (2004) foundational work, researchers generally hypothesize greater violence when groups have stronger incentives to compete. Conrad and Greene (2015, 547) concisely summarize a key mechanism underlying outbidding theory: "Since competition directly and indirectly threatens the resource base necessary to sustain the organization and ensure its effectiveness, it follows that terrorist organizations should make tactical choices in an effort to increase their share of resources within a competitive environment." When looking for evidence of outbidding, scholars therefore regress measures of violence on proxies for incentives to compete—e.g., the number of groups in a conflict—using time-series-cross-sectional data and test for a positive association.[2] Within this framework, Findley and Young (2012) find no relationship between competition and violence, Chenoweth (2010), Cunningham, Bakke and Seymour (2012), and Wood and Kathman (2015) find a positive relationship, while others find more conditional findings (Conrad and Greene 2015; Conrad and Spaniel 2021; Nemeth 2014).

Previous research designs face two substantial weaknesses when uncovering evidence either for or against outbidding theories, however. First, they require proxies for competitive incentives, but directly evaluating the strength of these proxies is difficult, especially when commonly used measures (e.g., number of terrorist groups) are likely confounded by other aspects of the conflict (e.g., state strength). Second, they exclusively focus on competition's *encouragement effect* on violence and ignore its *discouragement effect*.[3] Both are implications of competition in contests, however (Chaudoin and Woon 2018; Dechenaux, Kovenock

[1] For example, Fortna (2015) and Getmansky and Zeitzoff (2014) argue that terrorism can be ineffective in this regard. In contrast, Gould and Klor (2010) and Thomas (2014) find that terrorism can make governments or citizens more accommodating.

[2] There is some disagreement on how to measure competitive incentives and on whether to measure the extent or intensity of terrorism.

[3] For example, as Conrad, Greene and Phillips (2023, 12) put it, "[the outbidding] argument *is* that intergroup competition leads to more violence [emphasis added]."

and Sheremeta 2015). On the one hand, enhanced competition can *encourage* violence because, if one group becomes more competitive, others may fight harder to keep up. On the other hand, enhanced competition can *discourage* violence because, if one group becomes more competitive, others may recognize a lost cause and give up. This creates a feedback loop where even the most competitive group uses little violence because it anticipates no pushback. By associating outbidding only with an encouragement effect, previous research designs overlook the discouragement effect and how the two countervailing effects can wash out in the aggregate, thereby masking evidence of outbidding.[4]

In this paper, we show how scholars can estimate the effects of competition on violence and better quantify the degree to which outbidding explains terrorism data. Our key departure from previous work is the structural approach. Broadly, the goal is to construct a model, estimate its parameters and equilibrium from observed data, and study properties of the fitted model (Canen and Ramsay 2023). Doing so has three main benefits in the context of outbidding. First, we flexibly estimate groups' incentives to compete, thereby sidestepping the need for proxies. Second, we use the fitted model to quantify the substantive effects of competition on violence by asking counterfactual questions such as "what would happen if one group expected no violence from its rival?" and "how would violence change if a group's competitive incentives increase?" Third, it reveals how well outbidding fits the data because we can see if nonsensical parameter estimates arise and explicitly analyze model fit and comparison.

To do this, we focus on the canonical outbidding example: the rivalry between Hamas and Fatah. Narrowing the scope of the analysis has several benefits. Theoretically, in a two-group rivalry, we model outbidding as a dynamic contest over public opinion wherein each side uses terrorism to pull public opinion towards itself and away from its opponent in a tug-of-war fashion. This conceptualization results in a two-actor, one-state-variable model, thereby reducing the number of to-be-estimated parameters. Empirically, given the rivalry's length, we compile monthly survey data that record aspects of Palestinian public opinion from 1994 to 2018. The data provide fine-grained details on how Palestinians view the conflict and the two groups, which we use to measure the relative popularity of Hamas and Fatah. Substantively, because it is the canonical (and theory generating) example of outbidding (Bloom 2004; Jaeger et al. 2015), it is of first-order importance to understand whether the discouragement effect emerges in this rivalry. If such evidence exists, then work extrapolating to other environments should not treat the discouragement effect as a mere theoretical curiosity when looking for evidence of outbidding.

Our main result is that we identify and quantify two discouragement effects. First, we

[4]To be clear, we follow Kydd and Walter (2006) and Conrad and Spaniel (2021) and use "outbidding" to refer to a theory where groups use costly terrorism to increase their popularity relative to another group. This theory is consistent with either the discouragement or encouragement effect. Unlike past works, we do not assume that outbidding is only consistent with the encouragement effect.

compare the estimated equilibrium rates of terrorism to those from counterfactual scenarios in which each group never anticipates violence from its rival. Comparing how a group behaves with and without violence from its rival is one way to compare group behavior in competitive and noncompetitive environments, respectively. We find that competition from Hamas has an encouragement effect on Fatah's use of violence, where Fatah is 34% more violent in equilibrium than when it expects Hamas to never attack—which is expected in the outbidding literature. In contrast, we find that competition from Fatah can deter violence from Hamas. During the Oslo era between 1994 and 2001, Hamas is 4% less violent in equilibrium than when it expects Fatah to never use violence. That is, competition from Fatah depresses Hamas's use of violence even during the time when the two groups are publicly vying for support from the Palestinians—this is the unexpected discouragement effect. Moreover, this result is consistent with some qualitative literature arguing that the mid-to-late 1990s were a low-point for Hamas, where the group, despite maintaining a campaign of violence, struggled to overcome Fatah's popularity and was bordering on irrelevance (Kirchofer 2015; Natil 2015).[5] After the Oslo era, we again find an encouragement effect where Hamas uses more violence because of competition from Fatah.

Second, we conduct comparative statics that demonstrate how equilibrium rates of violence change as a group becomes more or less competitive, i.e., has stronger or weaker incentives to compete. Whereas the first set of counterfactuals fixes the behavior of one group, this second set illustrates how the behavior of both groups change as incentives to compete change. In our framework (and in other contest models), groups have stronger competitive incentives when they place greater value on their popularity, have smaller costs of attacking, or become more effective at using terrorism to attract support. We find that making Hamas more competitive along any of these three dimensions increases the probability that either group uses terrorism. This is the expected encouragement effect in the outbidding literature where increasing the competitiveness of an actor leads to an increase in violence for not only the group in question but all groups involved. If Fatah becomes more competitive along any of these dimensions, however, both groups' propensities for terrorism decrease. This is the unexpected discouragement effect of outbidding.

Our theoretical framework explains these results via asymmetric competition. Although we find that Hamas has both lower costs to terrorism and places higher value on its public support than Fatah, Fatah is more effective at increasing its support through attacks than Hamas. That is, attacks by Fatah result in larger pro-Fatah shifts in public opinion than the corresponding effects of Hamas attacks on pro-Hamas shifts.[6] Because Fatah is substantially more capable at moving public opinion with violence, if its incentives to compete increase,

[5] At the time, Kristianasen (1999) speculates, albeit incorrectly, that Hamas may soon be an irrelevant actor due to Fatah's relative popularity and standing.

[6] As discussed below, this finding is robust to additional time-varying controls, different codings of attacks, and allowing for the effectiveness of attacks to vary over time. The result still holds even when instrumenting group attacks with past weather conditions in the Gaza Strip and West Bank.

then the group is more willing to take on the immediate costs of violence to move popular opinion more quickly. Hamas cannot compete with Fatah's level of efficiency and reduces its use of terrorism. This creates an equilibrium feedback loop and decreases Fatah's propensity to attack as its rival becomes more nonviolent.

Although our major contributions are showing how to estimate the effects of competition on violence and demonstrating that discouragement effects appear in a real-world outbidding contest, our analysis also provides evidence on the strength of outbidding in the specific case. Morever, we do so in a way that does not require indirect proxies for competition or assuming away heterogeneous effects. First, outbidding implies restrictions on our model's parameters, e.g., groups should value increased popularity. We do not impose these restrictions, and our estimates satisfy outbidding's theoretical restrictions throughout our analysis and robustness checks. Second, we compare our model to a no-competition version where competition does not arise because either the groups do not care about popularity or attacks do not affect their popularity. We reject the no-competition model. Third, we compare our fitted outbidding model to an alternative tit-for-tat model, which we fit to the same attack data. Using a non-nested model fit test, we find that the outbidding model fits the data better. To be clear, we are not claiming that outbidding is the best explanation or explains all of the observed terrorism.[7] Instead, the exercise demonstrates that outbidding can be compared explicitly to other strategic explanations of terrorism when scholars adopt the structural approach.

Our paper has two broader implications for the conflict literature. First, it implies that reduced-form correlations between competition and violence, like those reported in time-series cross-section regressions, cannot falsify outbidding theories: a theory of outbidding is consistent with a positive, negative or null relationship between competition and violence. Moreover, these correlations risk hiding evidence of outbidding because the encouragement and discouragement effects run in opposite directions. Although the contest literature has theoretically characterized the conditions under which discouragement effects appear (e.g., Kirkegaard 2012; Konrad and Kovenock 2005), it is unclear whether encouragement or discouragement effects would dominate in any given case and how conflict scholars would know. Our contribution is to show that discouragement effects are a salient feature of outbidding amount terrorist groups.

Second, our paper provides a general methodological approach to studying the effects of competition in dynamic contests in and outside of International Relations. In intrastate conflict, outbidding also appears among separatist groups in Northern Ireland or militant leftists in Colombia. These cases could be used as straightforward applications of our structural approach. In the interstate setting, arms races can be cast as a country using military investments to favorably adjust its security environment vis-a-vis a rival (e.g., Fearon

[7]Indeed, we find some Hamas attacks in the mid-1990s that occur even when our model predicts low Hamas attack probabilities. These attacks were attributed to spoiling motives by Kydd and Walter (2002).

2011; Powell 1993). Applying our model to this context, the state variable is interpreted as relative military power and the actions are whether or not to invest in arming. With time-series data on countries' decisions to acquire arms and on the evolution of military power, scholars can estimate an identical dynamic contest and use similar counterfactuals to quantify the substantive effects of competition on the balance of power. Additionally, trade wars and major-power competition for influence and proteges can be conceptualized as a tug-of-war competition. More broadly, a growing political economy literature estimates contest-like models, but these are either one-shot games (Kang 2016; Kenkel and Ramsay 2023; Köning et al. 2017) or include only one long-term player (Iaryczower, Lopez-Moctezuma and Meirowitz 2021). Thus, our paper helps scholars study empirical contests in a wider array of scenarios.

2 Model

Hamas (H) and Fatah (F) compete over a countably infinite number of periods indexed by $t \in \mathbb{N}$. In our data, a period corresponds to a calendar month. Period t's interaction explicitly depends on a publicly observed state variable $s^t \in \mathcal{S}$ that measures the relative popularity of Fatah over Hamas among Palestinians.[8] The set of states $\mathcal{S} = \{s_1, s_2, \ldots, s_K\} \subseteq \mathbb{R}$ is finite with $K \geq 3$ equally spaced popularity levels where $k > k'$ if and only if $s_k > s_{k'}$. We say Fatah is relatively more popular in state s than in state s' if $s > s'$ and vice versa for Hamas. In other words, smaller (larger) states represent periods where Hamas (Fatah) is more relatively popular.

Within each period t, Hamas and Fatah choose whether to commit a terrorist attack ($a_i^t = 1$) or not ($a_i^t = 0$), where $i = H, F$ indexes the group.[9] Given an action profile $a^t = (a_H^t, a_F^t)$, per-period payoffs are $u_i(a_i^t, s^t; \theta) + \varepsilon_i^t(a_i^t)$. The term $\varepsilon_i^t \in \mathbb{R}^2$ is a vector of action-specific payoff shocks that are private information to group i, where $\varepsilon_i^t(a_i^t)$ refers to the (a_i^t+1)th element of vector ε_i^t. The shock $\varepsilon_i^t(a_i)$ is an independent and identically distributed (iid) draw from a standard type-one extreme value (T1EV) distribution.[10] The shocks account for unobserved factors temporarily affecting the costs and benefits of terrorism and ensure that choices within each period are stochastic.

[8]We focus on relative popularity because several theories of outbidding maintain an underlying assumption that the benefits are "primarily relative or positional—i.e., the value of the resources gained depends on how much of that resource the group's competitors possess" (Gibilisco, Kenkel and Rueda 2022, 9).

[9]We model actions as binary for two reasons. Theoretically, discrete-choice models have well-understood properties (Pesendorfer and Schmidt-Dengler 2008; Su and Judd 2012). Empirically, these groups rarely attack more than once month: Fatah attacks more than once (twice) a month in 2.7% (0.7%) of observations, and Hamas more than once (twice) a month in 26% (16%) of observations—see Figure A.1 in Appendix A.

[10]This assumption is imposed to induce easy-to-use logit choice probabilities over actions and is a common simplifying assumption in structural models (e.g., Crisman-Cox and Gibilisco 2018; Frey, López-Moctezuma and Montero 2021; Rust 1994).

The term $u_i(a_i^t, s^t; \theta)$ is the systematic component of group i's per-period payoff and consists of popularity benefits and attack costs:

$$u_i(a_i^t, s^t; \theta) = \underbrace{\beta_i \cdot s^t}_{\text{popularity benefit}} + \underbrace{\kappa_i \cdot a_i^t}_{\text{attack cost}}, \tag{1}$$

where $\theta = (\beta_H, \beta_F, \kappa_H, \kappa_F)$. Because $\beta_i \cdot s^t$ captures i's benefit from relative popularity level s^t, we expect $\beta_H < 0$ and $\beta_F > 0$, i.e., groups want more favorable public support.This is one incentive for groups to compete. Likewise, κ_i denotes i's *cost* of attacking, which is another competitive incentive, and we expect $\kappa_i < 0$. Note that these inequalities are theoretical expectations from the outbidding literature. One of our goals is to estimate these unobserved competitive incentives from the observed data. We do not impose these inequalities as *a priori* restrictions, but we explicitly test these hypotheses using the fitted model.

In contrast, outbidding theories do not offer explicit expectations about the relative magnitudes of β_i and κ_i across actors. It could be that $|\beta_H| > |\beta_F|$ because Fatah has outside support from Israel and the U.S., which means it might care less about local Palestinian support. A similar argument suggests $|\beta_H| < |\beta_F|$, however, because Hamas has outside support from Iran, Syria, and Qatar during this time frame. Likewise, while intuition suggests that Hamas has smaller attack costs given the stark differences in the groups use of violence (Figure A.1 in Appendix A), outbidding theories do not have explicit predictions about relative attack costs. The model accommodates either possibility—i.e., Hamas may have larger or smaller competitive incentives than Fatah on any dimension—and allows us to quantify the differences post-estimation.

The sequence of the game in period t is as follows.

1. Group i observes s^t and ε_i^t.
2. Groups simultaneously choose whether to attack $a_i^t \in \{0, 1\}$.[11]
3. Payoffs are accrued.
4. Transition to period $t + 1$.

As the game transitions from period t to $t + 1$, popularity evolves according to an AR-1 process with a mean that depends on the chosen actions and state. Given today's support and attack decisions (a^t, s^t), we define the mean of tomorrow's support s^{t+1} as

$$\mu[a^t, s^t; \gamma] = \gamma_0 + \gamma_1 \cdot s^t + \sum_i (\gamma_{i,1} + \gamma_{i,2} \cdot s^t) \cdot a_i^t. \tag{2}$$

[11] Although it is a standard assumption in the contest literature (e.g., Chaudoin and Woon 2018; Conrad and Spaniel 2021; Konrad and Kovenock 2005), simultaneous choice within a period is a simplification because, in order to estimate a sequential model, we would need to specify a particular group to move first within each period t. We cannot infer such an ordering from the observed data, however, because the group that attacks first may be different than the group that had the first opportunity to attack.

The term $(\gamma_{i,1} + \gamma_{i,2} \cdot s^t)$ represents group i's ability at using terrorist attacks to increase its support—what we call i's *effectiveness* of attacks, which is the third competitive incentive in the model.[12] Outbidding theories expect $\gamma_{H,1} < 0$ and $\gamma_{F,1} > 0$, that is, attacks from group i pull popular support in i's preferred direction. These inequalities are theoretical expectations but are not imposed in estimation. As with the payoff parameters, outbidding does not have explicit expectations about the relative magnitudes of $\gamma_{F,1}$ and $\gamma_{H,1}$ (i.e., about which group is more effective at using terrorism), but the model accommodates either possibility. Note that Equation 2 allows the effects of i's attacks (i.e., $\gamma_{i,1} + \gamma_{i,2} \cdot s^t$) to depend on the current popularity level s^t. *A priori*, it is not clear whether group i's attacks should be more or less effective as its popularity increases. On one hand, if its popularity is large, then its attacks may be more effective due to support from the local population, implying that $\gamma_{i,2} > 0$. On the other hand, if its popularity is large, then there is less of the population to be won over, implying that $\gamma_{i,2} < 0$.

In period $t+1$, the probability that $s^{t+1} = s'$ given action profile a^t and state s^t is $f(s'; a^t, s^t, \gamma)$. We specify f using a discretized normal distribution:

$$f(s'; a^t, s^t, \gamma) = \begin{cases} \Phi\left(\frac{s'+d-\mu[a^t,s^t;\gamma]}{\sigma}\right) - \Phi\left(\frac{s'-d-\mu[a^t,s^t;\gamma]}{\sigma}\right) & s' \in \{s_2, \ldots, s_{K-1}\} \\ \Phi\left(\frac{s_1+d-\mu[a^t,s^t;\gamma]}{\sigma}\right) & s' = s_1 \\ 1 - \Phi\left(\frac{s_K-d-\mu[a^t,s^t;\gamma]}{\sigma}\right) & s' = s_K \end{cases} \tag{3}$$

where Φ is the standard normal cumulative distribution function, σ is the standard deviation parameter, and $2d = s_2 - s_1$ is the distance between the equally spaced relative popularity levels. The parameters $\gamma = (\gamma_0, \gamma_1, \gamma_{H,1}, \gamma_{H,2}, \gamma_{F,1}, \gamma_{F,2}, \sigma)$ describe the transitions of the game, and we estimate them below. We choose this specification because γ can be estimated using standard techniques for continuous AR-1 models even though the model has a discrete state space (Tauchen 1986).

2.1 Equilibria

Given a sequence of states, actions, and payoff shocks $\{s^t, a_i^t, \varepsilon_i^t\}_{t=1}^{\infty}$, group i's total payoffs are $\sum_{t=1}^{\infty} \delta^{t-1} \left[u_i(a_i^t, s^t) + \varepsilon_i^t(a_i^t)\right]$ where $\delta \in (0,1)$ is a fixed, common discount factor. Discount factors are difficult to identify in dynamic discrete choice models (Abbring and Daljord 2020). Following Rust (1994) and others (e.g., Frey, López-Moctezuma and Montero 2021), we estimate the model at several discount factors and fix the discount factor to $\delta = 0.999$, which resulted in the highest log-likelihood.[13] This matches anecdotal descriptions

[12] We are using effectiveness in the context of outbidding. Of course, terrorism can have other dimensions of effectiveness in other environments, e.g., ability to hurt the government.

[13] We also show that our results are robust for $\delta \geq 0.975$. See Appendix I for details.

of the groups that highlight their long time horizons.[14]

Markov equilibria in discrete dynamic games with per-period private-information payoff shocks have a straightforward characterization.[15] Dropping references to time, let $v_i(a_i, s)$ denote i's net-of-shock expected utility from choosing action a_i in state s and continuing to play the game for an infinite number of periods. The vector $v_i = (v_i(a_i, s))_{(a_i,s)\in\{0,1\}\times\mathcal{S}}$ collects these values for each (a_i, s) pair. In other words, given a vector of expected utility values v_i and a vector of random shocks $\varepsilon_i = (\varepsilon(0), \varepsilon(1))$, group i chooses action a_i in state s if and only if

$$a_i = \underset{a_i\in\{0,1\}}{\operatorname{argmax}} \{v_i(a_i, s) + \varepsilon_i(a_i)\}.$$

Thus, v_i implicitly specifies a cut-off strategy for i, where i chooses to attack ($a_i = 1$) in state s if and only if $v_i(1, s) - v_i(0, s) > \varepsilon_i(0) - \varepsilon_i(1)$, where we sidestep the probability 0 event that $v_i(1, s) - v_i(0, s) = \varepsilon_i(0) - \varepsilon_i(1)$. Because $\varepsilon_i(0)$ and $\varepsilon_i(1)$ are iid draws from a standard T1EV distribution, i chooses a_i in state s with probability

$$P(a_i, s; v_i) = \frac{\exp\{v_i(a_i, s)\}}{\exp\{v_i(0, s)\} + \exp\{v_i(1, s)\}}\,. \tag{4}$$

Let g denote the joint density of ε_i. Group i's continuation value for state s is

$$\begin{aligned} V_i(s, v_i) &= \int \max_{a_i\in\{0,1\}} \{v_i(a_i, s) + \varepsilon_i(a_i)\}\, g(\varepsilon_i) d\varepsilon_i \\ &= \log\left(\exp\{v_i(0, s)\} + \exp\{v_i(1, s)\}\right) + C, \end{aligned} \tag{5}$$

where C is Euler's constant. The second equality in Equation 5 follows from McFadden (1978, 82) because g is the joint density of two iid standard T1EV random variables. Consider a profile $v = (v_i, v_j)$ of action-state expected utility values. Group i's iterative net-of-shock expected utility of action a_i in state s, denoted $\mathcal{V}_i(a_i, s, v; \theta, \gamma)$, is

$$\mathcal{V}_i(a_i, s, v; \theta, \gamma) = \underbrace{u_i(a_i, s; \theta)}_{\substack{i\text{'s payoff}\\ \text{today}}} + \delta \underbrace{\left[\sum_{a_j} P(a_j, s; v_j) \underbrace{\sum_{s'\in\mathcal{S}} f(s'; a_i, a_j, s, \gamma) V_i\left(s', v_i\right)}_{\substack{i\text{'s expected continuation value for}\\ \text{tomorrow's popularity given } a_j}}\right]}_{\text{iterated expectation over } j\text{'s action}}. \tag{6}$$

[14] A reporter describes it as follows: "It's sometimes shocking to sort of hear what their timeline is. And they'll say... that justice is on our side and that we're doing the right thing. And if we're not able to do it, maybe our children will do it or maybe our grandchildren will do it. But they have this very long-term view of where this is going." ("Why Hamas Keeps Fighting and Losing", May 2021, `https://www.nytimes.com/2021/05/26/podcasts/the-daily/gaza-hamas-israel-war.html`).

[15] Pesendorfer and Schmidt-Dengler (2008, Theorem 1) prove existence of Markov equilibria in a class of games that subsumes our game.

An equilibrium is a profile v that satisfies the following fixed-point condition:

$$v = \mathcal{V}(v; \theta, \gamma) \equiv \times_i \times_{(a_i,s)} \mathcal{V}_i(a_i, s, v; \theta, \gamma). \tag{7}$$

Equations 4–7 characterize equilibria as a system of $4K$ equations, where K is the number of relative popularity levels. In words, starting with i's net-of-shock, action-specific expected utilities, Equations 4 and 5 return i's choice probabilities and continuation values, respectively. Then, Equation 6 updates i's net-of-shock action-specific expected utilities, holding fixed i's continuation values and j's choice probabilities. An equilibrium is a fixed-point in Equation 7. In Appendix B, we consider a symmetric example, use Equation 7 to compute equilibria, and then study their substantive properties and comparative statics.

2.2 Remarks

Before proceeding, several remarks on the model are in order. First, because this is a model of outbidding, it explains variation in violence via intergroup competition and abstracts away from other motives for terrorism and from other nuances of conflict. This spartan approach is critical for our argument: outbidding produces heterogeneous relationships between competition and violence, and one such relationship is a discouragement effect where competition decreases violence. Furthermore, this discouragement effect appears in the canonical case of outbidding. Adding more moving pieces to the analysis only obfuscates this central result, which shows that outbidding is sufficient to produce a negative relationship between competition and violence; other strategic tensions are not necessary. Future work should consider the empirical strength of competing explanations for terrorism by developing and then estimating different structural models, which can be compared to ours through model-fit exercises. A necessary first step is to provide models of each theory and fit them to the same data. We start this process with outbidding. In Section 6, we illustrate how to make these comparisons by comparing the outbidding model to an alternative explanation.

Nonetheless, a structural analysis can provide *some* evidence about outbidding's ability to explain terrorism data. As discussed above, outbidding has specific theoretical expectations about each group's incentives to compete (captured by the signs of β_i, κ_i, and $\gamma_{i,1}$) that we treat as testable hypotheses and reject the null that they do not hold. In addition, we conduct a model fit exercise where we compare the fitted model to a nested no-competition model; we reject the no-competition model. More descriptively, we can also compare the attack probabilities in the estimated model to observed attacks to see what time periods the model explains poorly. Overall, outbidding can explain the dynamics of terrorist attacks quite well, although there exist some Hamas attacks in the late 90s that are more difficult to explain. Indeed, Kydd and Walter (2002) argue that some of these attacks were aimed to sabotage peace treaties (e.g., "spoiling"), a motive for terrorism which is outside the scope of outbidding.

Second, we do not model the decision of individuals in the local population who choose a group to support, a simplifying assumption that also appears in Conrad and Spaniel (2021) and structural models of dynamic elections (e.g., Iaryczower, Lopez-Moctezuma and Meirowitz 2021). Instead, individuals and their choices are captured by the functions μ and f, which describe how relative support evolves given the attack decisions of the two groups and their current popularity level. Rather than microfounding this behavior, we calibrate it to data by estimating the relevant parameters of interest, γ. Doing so allows us to sidestep additional assumptions detailing the decision of local individuals who may be myopic or adopt behavioral rules. In other words, our groups best respond to the behavior of their rivals given the patterns of public support, explored in previous work and estimated below.

Third, we use the terms of value, cost, and effectiveness to describe competitive incentives and thereby explicitly borrow phrasing from the contest literature as our model has similarities with dynamic battles (e.g., Konrad and Kovenock 2005). Conrad and Spaniel (2021) also use a contest model to study outbidding. Besides the structural approach, our key departure is twofold. First, we consider a dynamic environment whereas Conrad and Spaniel (2021) consider a static model. Second, because we are interested in studying the effect of competition using the version of the model most closely tethered to observables, we consider a fully asymmetric contest where competitive incentives can vary by actor. In contrast, Conrad and Spaniel's (2021) main predictions require certain symmetry assumptions. The differences are substantively important because the contest literature anticipates that the discouragement effect appears in asymmetric (e.g., Kirkegaard 2012) and dynamic contests (e.g., Konrad and Kovenock 2005). Thus, discouragement effects will be overlooked when focusing on symmetric, one-shot contests.

3 Data sources and measurement

Terrorism data are from the Global Terrorism Database (GTD) where we record terrorist attacks committed by Fatah/PLO and Hamas from January 1994 to December 2018.[16] The GTD records both suicide bombings, which are the focus of Bloom (2004) and Findley and Young (2012), but also other types of terrorism, e.g., rocket attacks, which are greater part of violence against Israelis in recent years (Getmansky and Zeitzoff 2014). Hamas engages in an average of roughly 1.5 attacks per month, while Fatah engages in an average of less than 1 attack per month—see Figure A.1 in Appendix A for details. To measure group i's attack decision in month t, we record a dummy variable indicating whether the group committed any terrorist attacks in that month.

[16] In Appendix H, we reestimate the model using different time frames; our results are stable across subsamples.

The model's state variable reflects the relative popularity of the two groups among Palestinians. To measure it, we treat relative popularity as a dynamic latent variable and use observed public-opinion variables as its indicators. To assemble the set of indicators, we use surveys from the Jerusalem Media & Communication Centre (JMCC N.d.) and the Palestinian Center for Policy and Survey Research (PCPSR N.d.). The JMCC publishes two to six surveys per year consisting of random samples of Palestinian adults. They conduct face-to-face interviews in randomly selected households from randomly selected neighborhoods throughout the West Bank and Gaza Strip; the subjects inside each home were selected using Kish tables. Each survey typically occurs over a few days but less than a week. The average sample size is 1,210, with a range between 1000 and 1920.[17] On average 63% of respondents are from the West Bank, although this varies between 60–73%. Given their rich data about Palestinian attitudes, these surveys appear in other studies (e.g., Clauset et al. 2010; Jaeger et al. 2012).

PCPSR (also known as Center for Palestine Research & Studies until July 2000) runs two to nine surveys per year. It generally uses a multi-step selection process where they randomly sample locations in proportion to the population from a list of all cities, towns, villages and refugee camps in the West Bank and Gaza Strip. Once locations are selected, they sample individual blocks and then individual households. Like the JMCC surveys, each survey here typically occurs over several days but less than a week. Sample sizes vary between 1,076 and 2,150 with a mean of 1,320.[18] West Bank respondents tend to make up about 60–67% (mean 63%) of the overall sample, with Gaza respondents making up the rest.

We search through every survey published by these centers between 1994 and 2018 to track Palestinian public opinion for both actors using three dimensions. The first tracks which political or religious group respondents trust most from the JMCC. The second asks which political party each respondent supports from the PCPSR. The third is similar and asks which party they intend to vote for in legislative elections from the JMCC. For each of these three questions we track the proportion of respondents who answer Hamas or Fatah.[19] These three questions are open ended. Appendix C contains more details on

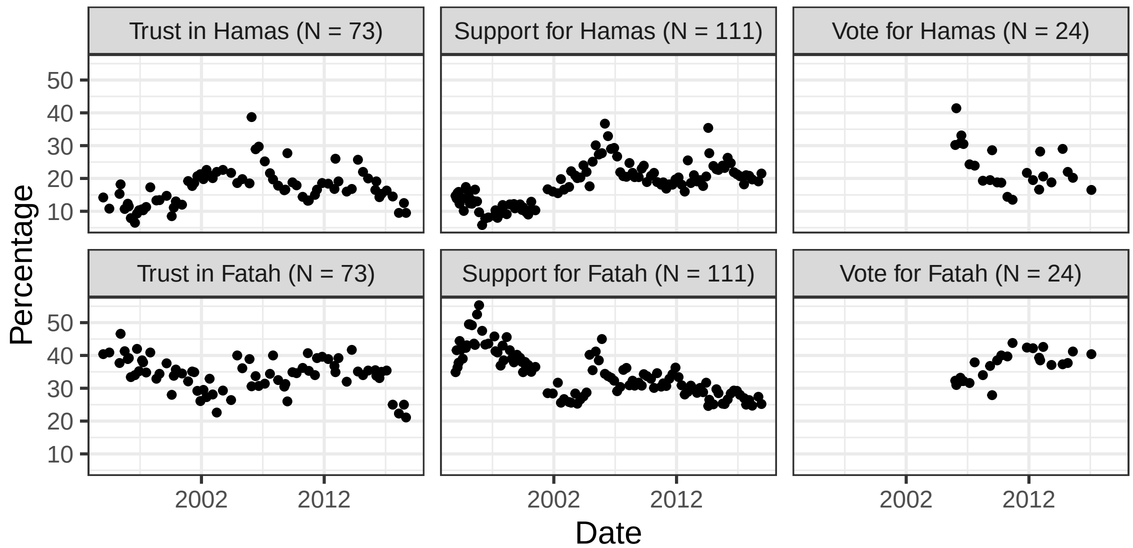
[17]In the majority (but not all) of polls, the JMCC breaks down answers by geography. In the West Bank, the average sample size is 768, with a range of 625 to 1,246. In the Gaza Strip, the average is 443, with a range of 342 to 674.

[18]In the West Bank and Gaza Strip the ranges are 664–1311 (mean of 850) and 390–695 (mean of 504), respectively. The surveys continue to report the results by region, but stop reporting the regional sample sizes in 2009.

[19]We also examine the total percentage of people saying they trust/support either group and how this varies over time. Regressing these totals on time, observed terrorism, and their interactions, we find that fitted values range from 51.2-51.4% for trust and 50-55% for support over our sample period. This indicates that the expected level of trust/support available to these two actors is fairly stable over time, with the estimated conditional mean shifting by only a few percentage points. The notable, but one-off, exception is at the end of the Second Intifada when there is a surge in Hamas support such that total support for both actors crosses 70%.

question wording and variation by geography.

Figure 1: Survey responses over time.



Note: First column tracks JMCC question "Which political or religious faction do you trust the most?" Second tracks PCPSR question "Which of the following political parties do you support?" Third tracks JMCC question "If Legislative Council elections were held today, which party would you vote for?" For each panel, N reflects the number of months when the question was asked.

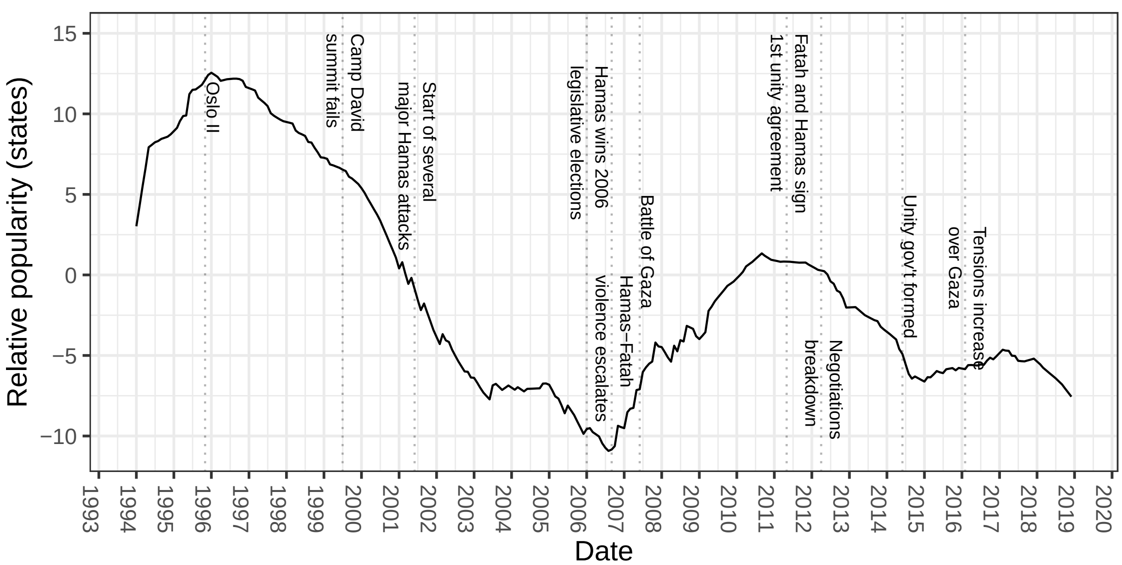
Figure 1 graphs responses to these six survey questions over time. These answers largely follow a basic trend where public attitudes towards the groups are inversely related. They show declining Fatah support during the 1990s and early 2000s with rising Hamas support. These trends level out a bit in the later years, with Fatah maybe regaining some support at the expense of Hamas. The surveys mostly correlate with each other in the expected directions (Table C.2 in Appendix C), which suggests that they can be collapsed onto one dimension. To do this, we use a dynamic factor model that transforms these polling questions into a continuous representation $\tilde{s}^t$ of the theoretical state variable s^t. See Appendix C for details.

Having produced the continuous state variable $\tilde{s}^t$, we assess its validity.[20] Figure 2 shows how the state variable evolves from 1994-2018. Fatah is favored in earlier period, where its relative popularity peaks during the 1996 Oslo II process (Jan. 1996 $\approx$ 12.5). Likewise, Hamas is at its most popular relative to Fatah in 2006 during the aftermath of the general election in which they took control of Gaza (Aug. 2006 ≈ -10.9). The mean of this variable is -0.87 (median of about -3) with a standard deviation of 6.59 (interquartile range of -6.02 to 4.13). The continuous state variable is easily mapped back onto the original surveys, such that, on average, a one unit increase in $\tilde{s}^t$ roughly corresponds to a

[20] All survey responses load onto the factor in the expected directions: pro-Hamas responses are more likely when $\tilde{s}^t$ is small, and pro-Fatah responses are more likely when $\tilde{s}^t$ is large. See Table C.3 in Appendix C.

0.9, 1.5, and 2 percentage point increases in net trust, support, and intention to vote for Fatah over Hamas, respectively.

Figure 2: Relative popularity of Fatah to Hamas over time.



Several important event are listed in Figure 2, providing context and face validity to the idea that this variable captures the relative ups and downs between the two groups. Notably, the late 1990s are typically regarded as an important inflection point in the relative standing of these two groups and that is clearly reflected here. Fatah sees its popular support erode as the peace process unravels. Furthermore, our measure has rich variation with substantial ups and downs that go undetected in existing measures of group popularity—e.g., Tokdemir and Akcinaroglu (2016) do not find popularity differences between Fatah and Hamas after 1997. Finally, in Appendix C, we demonstrate that our latent measure of relative popularity is robust to different model specification choices. The estimated state variables correlate highly (0.87-0.99) across specifications.

4 Estimation and identification

Following Rust (1994), we adopt a two-step estimation procedure where we first estimate how relative support evolves (γ) and then estimate the groups' payoff parameters (β, κ). To do this, first rewrite the AR(1) model in Equation 2 in terms of the continuous state variable $\tilde{s}^t$:

$$\tilde{s}^t = \gamma_0 + \gamma_1 \tilde{s}^{t-1} + \gamma_{H1} a_H^{t-1} + \gamma_{H2}(\tilde{s}^{t-1} \times a_H^{t-1}) + \gamma_{F1} a_F^{t-1} + \gamma_{F2}(\tilde{s}^{t-1} \times a_F^{t-1}) + \nu^t, \quad (8)$$

where a_F^{t-1} and a_H^{t-1} are binary indicators for whether Fatah and Hamas attack, respectively, and $\nu^t \sim N(0, \sigma^2)$.[21]

The first-step estimates are then used to construct the Markov transition probabilities, f. To discretize the continuous state $\tilde{s}^t$, we first define the lowest and highest (most Hamas and Fatah friendly) states as the bottom and top 2.5th percentiles of $\tilde{s}^t$. Discrete states between these extremes are defined at equally spaced intervals with distance $2d = 0.05$. In the baseline model, $K = 440$. We then map the continuous measure $\tilde{s}^t$ into the discrete measure s^t by finding the closest discrete state.[22] Let $\mu[a, s; \hat{\gamma}]$ be the fitted values from the first model (reported below in Table 1) for all possible combinations of action profiles with the discrete states. Plugging these fitted values and the estimated standard deviation $\hat{\sigma}$ into Equation 3 produces the transition probabilities.

Following Crisman-Cox and Gibilisco (2018) and Su and Judd (2012), we use constrained maximum likelihood estimation (CMLE) to estimate the payoff parameters $\theta = (\beta, \kappa)$. Specifically, let $Y = \left(s^t, a_H^t, a_F^t\right)_{t=1}^T$ denote the time series of observed data (relative popularity levels and attacks). We fix the transition probabilities using the first-step estimates, $\hat{\gamma}$, and the definition of f in Equation 3. The CMLE estimates $(\hat{\theta}, \hat{v})$ maximize the log-likelihood

$$L(v|Y) = \sum_{t=1}^{T} \left[\log P(a_H^t; s^t, v_H) + \log P(a_F^t; s^t, v_F)\right]$$

subject to the equilibrium constraint equations $v = \mathcal{V}(v; \theta, \hat{\gamma})$. For standard errors, we follow Silvey (1959) use the bordered Hessian to compute the variance-covariance matrix and use the two-step correction as described in Appendix F.

The game can have multiple equilibria. The CMLE allows for this multiplicity with its main identification assumption being that the data Y are generated from only one of these equilibria (Crisman-Cox and Gibilisco 2018; Su and Judd 2012). By treating the endogenous equilibrium quantities, v, as auxiliary parameters, the CMLE selects the values of v that best describe the data while still being an equilibrium of the model. In other words, the CMLE imposes an empirical selection rule: Choose the equilibrium associated with the highest log-likelihood. This process is a computationally feasible alternative to an approach that computes *all* equilibria at *every* optimization step and then always chooses the equilibrium that maximizes the log-likelihood at that optimization step (Su and Judd 2012, Proposition 1). The CMLE imposes this same empirical selection rule, but without the infeasible requirement of repeatedly enumerating all equilibria.[23]

Along with the assumption that one equilibrium is generating the data, three empirical

[21] Unit root tests suggest that the state variable $\tilde{s}_t$ is not stationary. However, because $\tilde{s}^t$ and $\tilde{s}^{t-1}$ are cointegrated, OLS will produce superconsistent estimates. We also fit the model using the Engle-Granger error correction method (ECM) for hypothesis testing.

[22] Appendix J shows that our estimates are robust to changes in the discretization process.

[23] For more technical details on the estimation method see Crisman-Cox and Gibilisco (2018); Su and Judd (2012).

moments pin down our parameters of interest. We estimate γ through observed variation in the state variable over time. We know that each action profile has a positive probability of being played at each relative popularity level given the distributional assumptions on ε_i^t, and that the probability of transitioning from level s to level s' is positive for all s and s'. As such, f can be estimated non-parametrically from frequency estimators with a sufficiently long time frame because, eventually, the equilibrium path will visit all states and all action profiles will be played in every state. When the transition probabilities are known, the payoff parameters are identified by their relationship to the equilibrium constraint $\mathcal{V}$ in Equation 6. A group's attack cost is identified through its baseline propensity to attack regardless of the state, and a group's value of public support is identified by the variation in its propensity to attack across states. To see why, note that when $\beta_i = 0$ (or $\delta = 0$), then Equations 1 and 6 imply i's probability of attacking is constant across states and only depends on its attack costs κ_i.

Formal identification of the payoff parameters θ follows from Pesendorfer and Schmidt-Dengler's (2008) Propositions 2 and 3. The former is a necessary condition stating that in this type of model, up to K payoff parameters per actor can be identified. We seek to estimate 2 parameters per group using $K = 440$ states, which satisfies the necessary condition. The connection between K and identification raises questions about how sensitive are the estimates to discretizing relative popularity; in Appendix J we show that our estimates are robust to both small and major changes in this process. The latter is a more involved sufficient condition for identifying θ that depends on the equilibrium choice probabilities, which we can verify given our estimated equilibrium—see Appendix E.

The discussion above details how the game's parameters, specifically, the groups' competitive incentives, can be identified given data generated from an equilibrium of the game. Another reasonable concern is how sensitive are the estimated incentives to forces outside the model, in particular, to interventions from Israel. Here, we anticipate that Israeli actions are more or less important depending on whether the incentive is group effectiveness or directly enters the groups' payoff functions. For the former, we can compare our baseline estimates of $\gamma_{i,1}$ to those in robustness exercises where we either control for Israeli actions or their proxies (e.g., number of Palestinian fatalities in the conflict or days since the last Israeli election) or instrument group attacks with rainfall. In Appendix D, we show that our estimates of groups' attack effectiveness are stable across specifications.

For the latter, the analysis is murkier because we are unable to conduct such robust exercises. If we wanted to include Israeli interventions when estimating the groups' value of support and cost of attacking, then we would need monthly level data on actions taken against the individual groups during our time frame. Furthermore, we would need to either estimate how the these actions evolve according to relative popularity levels and attack decisions or explicitly model the Israeli government as a third strategic actor. Given the scarcity of high-frequency data recording how Israel responds to individual groups and that outbidding theories generally treat governments as "tangential" (Kydd and Walter 2006), we think that the appropriate first step is to structurally estimate an outbidding model without government interventions. Nonetheless, we anticipate that the groups' costs of attacking include both their upfront costs of attacks (e.g., obtaining explosives) and the strategic backlash from the Israeli government (e.g., border walls and airstrikes). In addition, if Israeli interventions are aimed at reducing the likelihood of attacks, then these interventions should target groups precisely when the tug-of-war predicts a high attack probabilities. Thus, the observed probability of attacks would appear flatter as a function of relative popularity than in a world without interventions. This would attenuate our estimates of the groups' values of support because these are identified by variation in changes in relative attack probabilities as a function relative popularity.

5 Parameter estimates

Table 1: Regressing relative popularity (state variable) on terrorist attacks.

	Dependent variable:	
	State	Δ State
	AR(1)	ECM
Hamas attack, $\gamma_{H,1}$	−0.21	−0.21
		(0.04)
Fatah attacks, $\gamma_{F,1}$	1.12	1.04
		(0.05)
Lag state, γ_1	1.00	
Δ Lag state		0.33
		(0.04)
Hamas attacks × lag state, $\gamma_{H,2}$	0.01	0.002
		(0.01)
Fatah attacks × lag state, $\gamma_{F,2}$	0.03	0.01
		(0.01)
Constant, γ_0	−0.02	−0.01
		(0.02)
T	299	298
adj. R^2	0.999	0.721
σ	0.216	0.183

Note: Newey-West standard errors in parentheses. No standard errors are reported for the AR(1) model due to unit root.

Table 1 shows the first-stage estimates and demonstrates that attacks by Fatah and Hamas move the state space in the expected direction. Recall that estimates of $\gamma_{i,1}$ reflect each group's effectiveness at using terrorism to shift public support towards itself and away from its rival. In months when Hamas attacks, their relative popularity improves by an average of about 0.11–0.29 in the following month depending on the current support $\tilde{s}^t$. Likewise, when Fatah attacks, they can expect their relative popularity to improve by about 0.86–1.4 on average. As mentioned above, the scale of $\tilde{s}^t$ can be roughly compared with the net level of trust in Fatah over Hamas, so on average, these magnitudes roughly reflect shifts in net levels of trust for Fatah over Hamas.[24] Both of these effects are statistically significant in the ECM model. These results provide evidence that groups are capable of outbidding and that acts of terrorism carry popularity benefits to the group, which supports results from Jaeger et al. (2015). Likewise, these results support findings from Polo and González (2020) who find that terrorism can be used to build support among a civilian audience, particularly when the audience is well-defined along ethnic or religious lines.

In addition, we find that Fatah's use of terrorism more effectively increases pro-Fatah support than Hamas's use of terrorism increases pro-Hamas support. Specifically, we reject the hypothesis that the groups are equally effective at moving public opinion ($H_0 : \gamma_{H,1} + \gamma_{H,2} \cdot s + \gamma_{F,1} + \gamma_{F,2} \cdot s = 0$) at every level of relative popularity s using the estimates and Newey-West variance matrix from the ECM model.

One possible explanation is that, as the more pro-peace actor, attacks by Fatah provide more information to the public. In other words, attacks from Hamas are expected and do little to adjust public opinion. For Fatah, attacks are more surprising, and thus the public's beliefs about how committed Fatah is to the Palestinian cause adjust more dramatically after an attack. As such, even though attacks demonstrate the resolve of both groups, Fatah receives a larger boost in public opinion. This explanation is consistent with our parameter estimates, but it is, of course, a conjecture because it involves assumptions about the Palestinian population that we deliberately did not microfound above. Nonetheless future outbidding studies should consider the population side of the outbidding process and better identify why and when we observe asymmetries in the response to terrorism.

In Appendix D, we show that these relationships are not driven by omitted economic and political factors, e.g., unemployment, Palestinian attitudes toward violence, the Second Intifada, Israeli election timing, or Palestinian fatalities from Israeli forces (which is one proxy for government actions). We also find no evidence that the groups are becoming more or less effective during our time frame (see Table D.3). Overall, the relationships between attacks and shifts in public support are largely unchanged in either direction or magnitude across model specifications. We also consider alternative measures of attacks. Even when we measure violence using attack counts, fatalities, or fatalities per attack, we find Fatah

[24] These numbers can be multiplied by 1.5 or 2 to translate them into the average effect of terrorism on net support and net voting intention, respectively.

is more effective than Hamas (Table D.4). Finally, we study plausibly exogenous variation in attacks driven by extreme rainfall shocks in the Gaza Strip and the West Bank.[25] The results illustrate that our baseline estimates of $\gamma_{F,1}$ and $\gamma_{H,1}$ in Table 1 are similar in size and magnitude to those from an instrumental variables analysis, although we are hesitant to over interpret these results. See Appendix D.3 for details.

Table 2: Payoff estimates.

		Standard Errors	
	Estimates	BH	Two-step
Hamas value of popularity, β_H	-0.0071	0.0042	0.0056
Fatah value of popularity, β_F	0.0005	0.0003	0.0004
Hamas attack cost, κ_H	-0.95	0.23	0.28
Fatah attack cost, κ_F	-2.46	0.28	0.40
Log-Likelihood		-278.20	
T		300	

Note: Bordered-Hessian (BH) standard errors and two-step corrected standard errors

Table 2 presents estimates for the values of popularity and costs of attacks. The sign on each estimate is in the expected direction from outbidding theory and are statistically significant at conventional levels (one-sided tests). Both actors like being relatively more popular than their opponent, i.e., Hamas most prefers s_1 and Fatah most prefers s_K. It may be concerning that the β_i estimates are quite close to 0, but we reject the null hypothesis that both β_i estimates are 0. Furthermore, as shown in the Appendix K, we find that the estimates have strong impacts on the equilibrium attack probabilities despite their seemingly small magnitudes. Interestingly, Hamas values its support more than Fatah with $|\hat{\beta}_H|$ being an order of magnitude larger than $|\hat{\beta}_F|$. As mentioned above, one possible explanation for this could be that Fatah has more support from outside actors to consider than Hamas. While this is explanation is consistent with our parameter estimates, our analysis cannot rule out others.

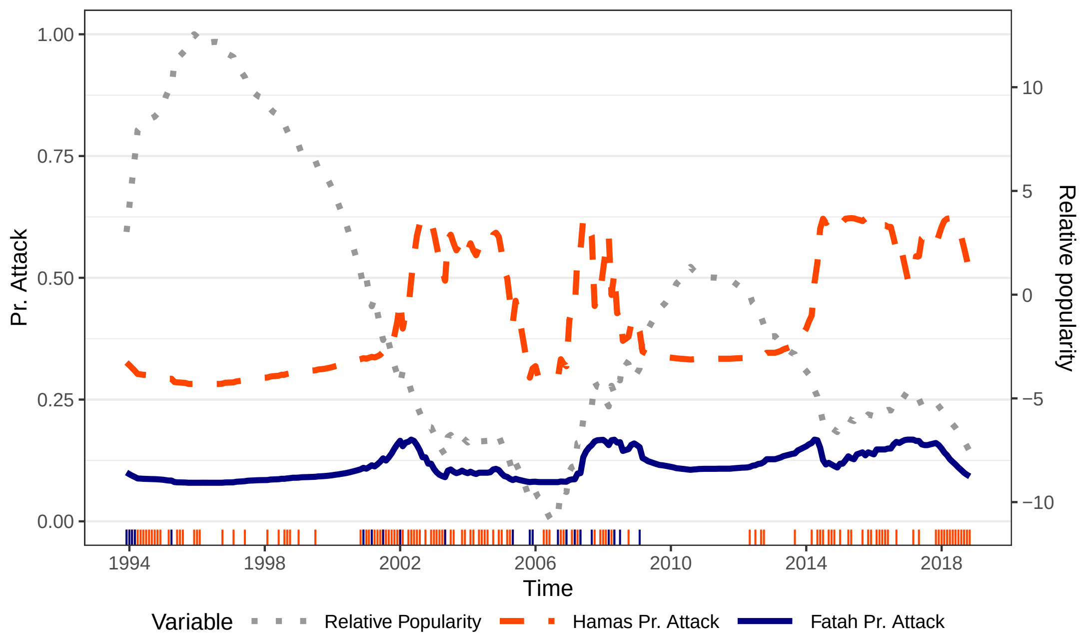
Intuitively, we find that terrorism is less costly for Hamas than Fatah, a finding which likely has several potential explanations. First, it could reflect different preferences for violence across the two groups, where members of Hamas have stronger preferences for terrorism than members of Fatah. Second, Hamas has made a concerted effort to build up its capacity for violence by developing infrastructure to acquire weapons and better train its members. As such, the group would find it less costly to engage in violence than Fatah which has devoted more resources to governance and engagement with the Israeli and

[25] Köning et al. (2017) pursue a similar approach when studying groups' use of violence in the Second Congo War.

U.S. governments. Both explanations fit with the historical record, which typically depicts Hamas as a more extreme actor while Fatah is a more practical political entity.[26]

Beyond the face validity of the point estimates, we consider the robustness of estimates in Table 2 in several Online Appendices. In Appendix F, we consider a sensitivity analysis to demonstrate that they are stable across a range of plausible first-stage estimates. In Appendix H, we consider several shorter time frames that represent potential starts, stops, or change points in the Hamas-Fatah rivalry, e.g., ending the data with the 2011 coalition agreement or starting in 1997, which is the first year included in Bloom (2004). In Appendix J, robustness to how we discretize our measure of relative popularity; are results are stable even with a small number of states, i.e., $15 < K \leq 22$. A discussion of model fit and alternatives is in the next section.

Figure 3: Estimated equilibrium probability of attacking over time.



Note: Horizontal axis denotes sample months/periods. Left vertical axis is the estimated probability that i attacks in month t, i.e., $P(a_i = 1; s^t, \hat{v}_i)$ where s^t is the observed relative popularity level in period t and $\hat{v}_i$ is estimated from the CMLE. For reference, s^t is also plotted on right vertical axis. The rug plot indicates observed attacks.

Finally, Figure 3 graphs the groups' estimated attack probabilities over time, i.e., $P(a_i = 1; s^t, \hat{v}_i)$. In addition, we also graph the relative popularity level s^t over time on the second horizontal axis for reference. Notice that Hamas has a higher probability of attacking than Fatah regardless of its relative popularity. Averaging over the observed states, Hamas attacks with probability 0.42 and Fatah with probability 0.11. This maps onto our estimates.

[26]Fatah officially renounced terrorism as part of its push to be recognized as a legitimate political actor, so attacks likely carry additional reputational costs for violating this pledge. Schanzer (2003) notes that this additional cost as a fundamental constraint on Fatah's abilities to respond violently when Hamas' popularity was increasing during the "Roadmap to Peace" era.

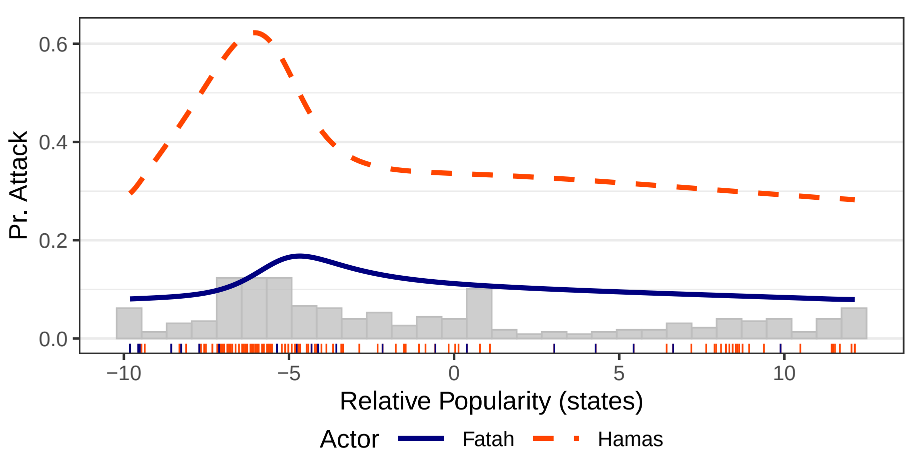
Hamas cares more about its popularity than Fatah, and it has a comparatively smaller attack cost although Fatah more effectively uses terrorism to increase its support. In addition, terrorism is particularly prevalent when Hamas is relatively popular, specifically during the Second Intifada and after the group wins legislative elections in 2006.

6 Model fit and comparison

Before considering the substantive implications of the estimated outbidding model, we consider how well it describes the data, both on its own terms and in comparison to alternative theories. In this section, our goal is not to test a particular causal hypothesis implied by outbidding or alternative theories, but rather to demonstrate the validity and usefulness of the model when explaining variation in the observed terrorism data.[27]

For the first exercise, recall that the estimated competitive incentives match the direction posited by outbidding theories, i.e., attacking is costly, groups value support, and attacks increase relative support. These restrictions were not imposed during estimation, and we would be skeptical of outbidding's ability to explain the data if they did not hold. For example, how could outbidding be a consistent theoretical explanation if groups wanted to become less popular? We can also examine the states in which the model predicts attacks well; Figure 4 does so visually. Ideally, we should see more attacks when the equilibrium choice probabilities are higher all else equal. For the most part, this is true: observed attacks fall mostly when relative popularity s is between -7 and -3 where the equilibrium choice probabilities peak.

Figure 4: Estimated equilibrium attack probabilities as a function of the state.



Note: Estimated probability that each group attacks as a function of relative popularity. The horizontal axis includes a rug plot of observed attacks and a histogram of the observed states s^t, where gray bars illustrate the density of the observed states.

[27] Crisman-Cox and Gibilisco (2018) and Kenkel and Ramsay (2023) conduct similar exercises to demonstrate the validity of their structural models of interstate crisis escalation.

Nonetheless, the data show a cluster of Hamas attacks around $s \in [8, 9]$ where attack probabilities are smaller. These attacks are difficult to attribute to outbidding as Hamas was close to its nadir of popularity, and the estimated attack probabilities suggest that, given the relatively few number of periods, this many attacks is relatively unlikely. As such, it seems reasonable to suspect that another theory of terrorism may better explain these attacks. Kydd and Walter (2002) argue that some of these attacks were part of an attempt by Hamas to undermine or "spoil" the Oslo process and drive a wedge between Fatah and Israeli negotiators. If these attacks are indeed more associated with spoiling and less attributable to outbidding, then it is not surprising that they stand out in Figure 4. This analysis highlights an advantage of this structural approach as it allows us to easily identify observations that do not easily fit the theory's predictions.

Moreover, it raises a question: can other theories of terrorism better explain the data? It is well beyond the scope of this paper, or possibility to be blunt, to consider and adjudicate among all theories of terrorism. Indeed, we think the field's understanding of the strategic forces behind terrorism will advance if scholars construct competing models of terrorism from different theories and estimate these models on the same data. Doing so, would allow for specific comparisons about how well the models and their associated theories explain variation in observed terrorism. Given the lack of previous structural models of terrorism, we have no obvious prior competing model for comparison. As such we create some alternative structural models and acknowledge that, until more models are available, our comparison models are inherently ad hoc. We look forward to future scholars creating and estimating competing models that can be horse-raced against an outbidding model.

The first alternative model is a null model where there is no-competition between groups either because groups cannot or do not care to compete with each other for popularity. We can nest such a model within our outbidding model by assuming $\gamma_{F,1} = \gamma_{F,2} = \gamma_{H,1} = \gamma_{H,2} = 0$. With this assumption, we cannot identify β. The only parameters left to fit the no-competition model are κ_F and κ_H, i.e., groups are attacking without reference to relative popularity and are only attacking due to static incentives. Because this alternative model is nested, it can be compared to the main model using a standard likelihood ratio test. As shown in Table 3, we reject the null hypothesis that the no-competition model fits as well as the main model.

The second alternative is a non-nested model based on tit-for-tat retaliation.[28] For this model, when group i chooses to attack ($a_i^t = 1$) or not ($a_i^t = 0$) in each period, we make the following assumptions:

[28]We choose this model because it is (i) a dynamic model, so we can use similar tools to characterize equilibria and estimate its parameters; (ii) supported by news articles and scholarly work (e.g., Brown 2012; Johannsen 2011), and (iii) an alternative explanation for competition suggested Michael Joseph, whom we thank for this suggestion. It has the added benefit that it has the same number of parameters entering each actor's utility function as the baseline model.

1. The new publicly observed state variable $r^t = (r_F^t, r_H^t) \in \{0,1\} \times \{0,1\}$ is a two-dimensional variable that records whether each actor attacked in the previous period, with $r_i^t = 1$ denoting group i attacked in period $t-1$.

2. The systematic utility function for group i is now

$$u_i(a_i^t, r^t; \tau, \kappa) = a_i^t \cdot (\underbrace{\kappa_i}_{\substack{\text{baseline}\\\text{cost}}} + \underbrace{\tau_i \cdot r_{-i}^t}_{\substack{\text{retaliaiton}\\\text{benefit}}}).$$

   Here, κ_i is the baseline cost of attacking, and τ_i is the additional benefit or cost a group receives when attacking in response to a previous attack from its rival. Collect these parameters into vectors $\kappa = (\kappa_H, \kappa_F)$ and $\tau = (\tau_H, \tau_F)$.

As in the baseline model, we assume that groups' per-period payoffs depend on a privately observed state variable $\varepsilon_i^t = (\varepsilon_i^t(0), \varepsilon_i^t(1))$ that represents action-specific payoff shocks and is distributed iid standard T1EV. As above, this tit-for-tat model is a discrete dynamic game, so we can use techniques almost identical to those in Section 2.1 to characterize Markov equilibria, except with appropriate changes to the utility functions and the state transitions, which are now deterministic as $r_i^t = a_i^{t-1}$. Moreover, we can use the CMLE to fit the model to the same GTD data to estimate κ and τ.[29] The goal is to compare how well this model explains the attack data versus our outbidding model.[30]

The point estimates from the tit-for-tat model are presented in Appendix G.1, where they are all in the expected directions for a tit-for-tat theory, i.e., attacking is costly but groups have an additional benefit if they attack in response to their rival. Comparing the tit-for-tat model to the outbidding model requires a non-nested model test. We use Clarke's (2007) test, which is a comparison of "point-wise" log-likelihood values—i.e., comparing the CMLE log-likelihood over the actions for each time period across models. The null hypothesis is that the two models are equally good; we reject this null in favor of the one-sided alternative that the outbidding model better fits the data. The test results are shown in Table 3. Overall, we conclude that the outbidding explains the data better than a competing tit-for-tat model.[31]

[29] Because transitions are deterministic, we do not need to estimate how the state variable evolves according to past actions and states. As above, results from Pesendorfer and Schmidt-Dengler (2008) imply identification. In fact, our restrictions on per-period payoffs in the tit-for-tat model match those used in the example from Pesendorfer and Schmidt-Dengler (2008, 913; Eq. 16-17).

[30] To code r^1, we need data on the group's use terrorism from December 1993, which is missing in the GTD, so we use Acosta and Ramos's (2017). Additionally, we set the discount factor δ to 0.999 to match the main model, but the model fit and point estimates are unchanged for nearly any $\delta > 0$ we try.

[31] Appendix G contains additional information about model fit. It also includes a comparison of the outbidding model to reduced-form results from a vector autoregression (VAR) model.

Table 3: Comparative model tests.

Alternative model	Test	Null distribution	Statistic	p value
No-competition	Likelihood ratio	$\chi^2(6)$	279.9	< 0.01
Tit-for-tat	Clarke's test	Binomial$(300, 0.5)$	182	< 0.01

7 Substantive effects of competition on violence

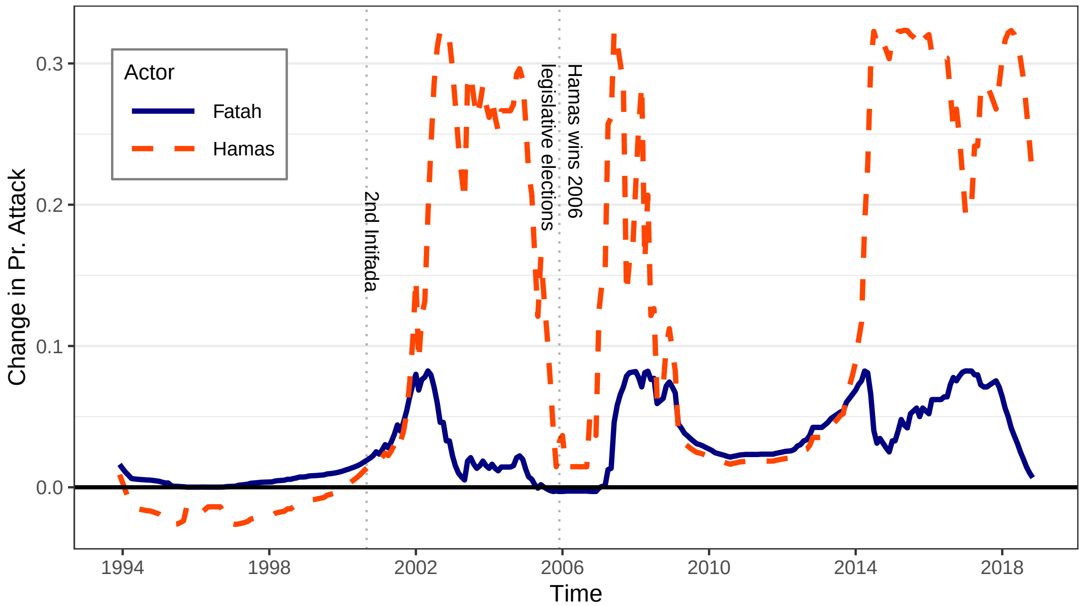
What is the substantive effect of competition on violence? Does heightened competition encourage or discourage violence? Answering these questions absent a structural analysis is difficult because raw attack rates—even changes in attack rates—cannot be used as evidence for either deterrent or encouragement effects. If we see a group using violence quite frequently (or infrequently) in a given time frame, then the pattern could be explained by small (or large) attack costs, the equilibrium path visiting states in which a group is likely (or unlikely) to use violence, or merely small sample bias arising because groups' decisions are stochastic. Instead of interpreting the data atheoretically, we use the fitted structural model to quantify how a group's use of violence *changes* as competition *changes*. To do this, we warp different aspects of competition in the fitted model and record how its predictions concerning the groups' use of violence would change in response. In other words, we quantify the effects of competition on violence in the version of the outbidding model most closely tethered to the data.

First, we compare how a group behaves with and without violence from its rival. That is, would Fatah use more or less violence if Hamas did not engage in terrorism and vice versa? Specially, we compare group i's estimated equilibrium probability of attacking (in Figure 3) to the probability of attacking in group i's single-agent problem, i.e., i's predicted use of violence if it expects its rival to never attack. Subtracting the latter from the former is one way to quantify the effect of competitive behavior on violence where the equilibrium attack probabilities represent violence in a competitive environment and the single-agent attack probabilities are from a noncompetitive environment. Figure 5 graphs these differences over time given the observed relative popularity s^t. Positive values indicate a positive effect of competition on violence, i.e., a group's equilibrium probability of attacking is larger than its probability of attacking in its single-agent problem. Negative values indicate a negative effect.[32] Thus, one interpretation of the figure is that the value in month t with popularity level s^t indicates the effect on group i's immediate attack probability if group $-i$ were to stop using violence in all future periods.[33]

[32] Figure A.2 in Appendix A graphs the difference in attack probabilities as a function of relative popularity levels.

[33] Rather than showing evidence either for or against outbidding, Figure 5 shows evidence of encouragement effects (positive numbers) or discouragement effects (negative number) for the two groups in different time periods.

Figure 5: Effects of competitive behavior on violence.



Note: For each month t (horizontal axis), we compare group i's equilibrium probability of terrorism to the probability that would arise if i expects its rival to never use violence, by subtracting the latter from the former given the observed state s^t. Positive values indicate that competition increases violence by group i in period t with state s^t; negative values indicate that competition decreases violence by group i.

For Fatah, the values are entirely positive indicating that Hamas encourages Fatah to use more violence than it would absent competition. On average competition from Hamas increases Fatah's use of violence by 34% from the counterfactual noncompetitive environment. This is the expected encouragement effect of competition on violence from the outbidding literature. Table 4 decomposes the effect over three time periods. It shows that Fatah's propensity for terrorism increases by about 3 percentage points due to competition from Hamas, especially after the start of the Second Intifada.

For Hamas, however, the story is different as heterogeneous effects exist. Competition from Fatah depresses Hamas's use of violence during the Oslo era, although we find a positive effect during and after the Second Intifada. Table 4 indicates that during the Oslo-era period, Hamas's propensity for terrorism would increase by about 1 percentage point in the absence of competition from Fatah on average. This point estimate represents an average over this period. If we consider the largest monthly effect, then we would predict a 9% increase in Hamas attacks if Fatah committed to never use violence. Put differently, this corresponds to a 4–5% reduction in violence from Hamas during Oslo lull compared to its counterfactual single-agent problem where Fatah never attacks. This is the discouragement effect of competition on violence where a group uses less violence in the competitive environment than in a noncompetitive one. Substantively, this change implies about 2–3 more months with Hamas terrorism in the counterfactual world versus in the observed data. While this is a relatively small effect, the potential devastation and loss of life associated with any given attack (particularly from Hamas) means that it is likely to be substantively meaningful.

These estimates suggest a competition-based explanation for the Oslo lull. Specifically, the popularity of the peace process during the 1990s boosted Fatah's standing among the Palestinian population. Figure 1 shows that Fatah frequently dominates Hamas in terms of trust and support by 30 percentage points during this time frame. Accordingly, relative popularity is overwhelmingly in Fatah's favor relative to the rest of sample (see Figure 4). As such, although Hamas has some incentives to use violence—it wants to pull public support away from Fatah—it also knows that the competition is very lopsided in Fatah's favor. Furthermore, Fatah is also more effective at using violence to increase its popularity, further depressing Hamas's use of violence.

Moreover, this theoretical account has anecdotal support in some contemporary understandings of the conflict. As Kristianasen (1999) writes, "[w]hile the Oslo agreement consecrated Hamas's role as a new national resistance to Israel, it ushered in a reality that progressively would tie the movement's hands" (1999, 20). They go on to argue that Hamas had issues remaining relevant during parts of this period due to Fatah's popularity and that delays and discontentment with the peace process (i.e., negative shocks to Fatah's public approval) were the main drivers of Hamas' ability to remain relevant. Indeed, during the mid-to-late 1990s, Hamas was largely operating underground and faced a resource and support shortage (Natil 2015, 38). Baconi (2018, 34) affirms this understanding, noting that "[b]y the end of 1997, the pressure Hamas was under meant that its suicide operations began to recede as it reverted to focusing on social infrastructure." While Hamas still pulled off several high-profile attacks during this time, its overall public support was low enough that it was unclear to contemporary observers if the group would continue to be a relevant actor (Kristianasen 1999, 33-4).

Of course, this explanation is not the only one for this period of Fatah-Hamas interactions. As mentioned above, the model does not include other key aspects of this relationship like efforts by Hamas to sabotage the peace outside of a desire to gain local support. However, this historical record does lend credence to the idea that Hamas may have been deterred by Fatah's popularity during this period as contemporary writers and conflict historians both acknowledge that Fatah's popularity during this period placed Hamas had a notable affect on Hamas' strategic calculus.

As shown in Figure 5 and Table 4, the presence of a rival terrorist group can depress violence. Although some outbidding studies argue that increasing the number of terrorist groups—a common proxy for competitiveness—can decrease violence, their underlying mechanisms do not appear in this setting. For example, Nemeth (2014) argues that increasing the number of ideologically similar groups should decrease violence through free-riding dynamics. Hamas and Fatah are generally seen as ideologically opposed, however, and

Table 4: Average effect of competitive behavior on violence in three eras.

	Jan. 1994 to Sep. 2000 Oslo era	Oct. 2000 to Jan. 2006 2nd Intifada	Feb. 2006 to Dec. 2018 post-2006 election
Hamas	−0.01	0.18	0.15
	(0.001)	(0.01)	(0.01)
Fatah	0.005	0.03	0.04
	(0.0004)	(0.002)	(0.001)

Note: Average difference between equilibrium and single-agent attack probabilities from different eras with standard errors in parentheses.

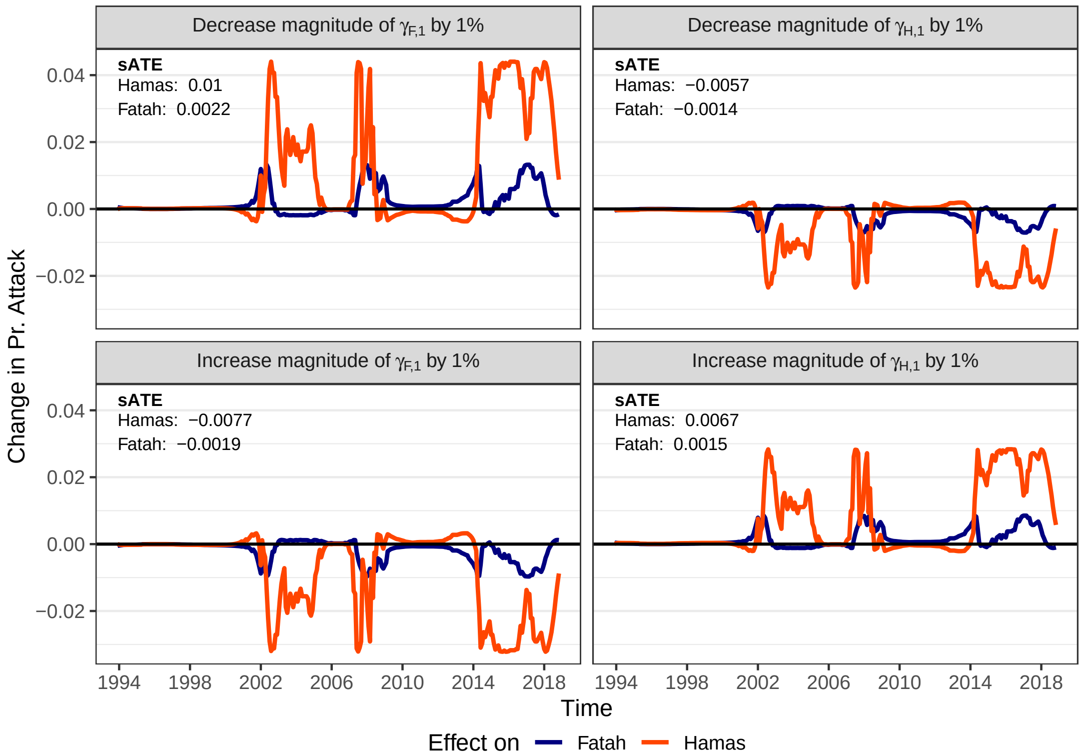
there are no free-riding incentives in the model. Another example is Conrad and Spaniel (2021) who argue that the government may change its demands in response to a large number of groups, leading to a negative correlation between terrorist group numbers and violence. Our results demonstrate that endogenous government demands are not necessary for competition to have a negative effect on violence.

Second, we examine how groups' competitive incentives affect their attack probabilities. For example, how would overall violence levels change if group i became a more effective outbidder, i.e., $\gamma_{H,1}$ becomes more negative or $\gamma_{F,1}$ becomes more positive? Whereas the first counterfactual quantifies the effects of competitive behavior on violence, this exercise illustrates the effects of competitive incentives on violence. To do this, we fix the transition parameters estimated from Table 1, the payoff parameters in Table 2, and the estimated equilibrium quantities. For each group i, we then change how effectively i can boost its popularity through terrorism by increasing and decreasing the magnitude of $\gamma_{i,1}$ by 1%. As the effectiveness of attacks changes, the equilibrium probabilities of attacks will change as well. Recall that $\gamma_{i,1}$ reflects the effectiveness of i at using terrorism to shift relative public opinion. An increase or decrease in $\gamma_{i,1}$ may reflect a change in tactics that the public may find more or less distasteful.

Because multiple equilibria can exist, we cannot just vary $\gamma_{i,1}$, compute a new equilibrium, and compare choice probabilities under the old and new parameter values. Doing so would not guarantee that the new equilibrium bears any resemblance to the estimated one. Indeed, it may be possible to change equilibrium behavior even though $\gamma_{i,1}$ does not change by changing the selected equilibrium. To ensure that the counterfactuals fix the equilibrium that is selected by the data in the CMLE, we use a procedure from Aguirregabiria (2012) and Crisman-Cox and Gibilisco (2018) that uses a homotopy method to map equilibria as locally continuous functions the parameters. Appendix L contains the details.

Figure 6 graphs theses differences given the change in $\gamma_{i,1}$ and observed state s^t. Positive values indicate that violence from group i in observed state s^t increases in the counterfactual scenario, whereas negative values indicate that violence decreases. As above, one interpretation of the figure is that values in month t with popularity level s^t indicate the effect on the groups' immediate attack probabilities if i were to exogenously become more or less competitive.

Figure 6: Relationship between terrorism and effectiveness of attacks.



Note: In each panel, we increase and decrease the magnitude of $\gamma_{i,1}$ for $i = H, F$ from its estimated value by 1%; all other parameters are held constant at their estimated values. We use a procedure from Aguirregabiria (2012) to account for the potential presence of multiple equilibria—see Appendix L for details. Incentives to compete are greater when $\gamma_{i,1}$ is larger in magnitude. The horizontal axis denotes the period/month t. The vertical axis is the difference between equilibrium attack probabilities (Figure 3) and counterfactual attack probabilities given the change in $\gamma_{i,1}$ and observed state s^t. Positive (negative) values indicate that violence by group i increases (decreases) in the counterfactual.

Focusing on the effects of Hamas's competitive incentives, we find evidence of outbidding's expected encouragement effect: when Hamas has greater incentives to compete, violence by both groups increases. We estimate that a 1% increase in Hamas's effectiveness results in a 1 percentage point increase in the frequency of terrorism by Hamas and a 0.1 percentage point increase in the frequency of terrorism by Fatah. On average, this implies Hamas would increase its use of violence by 2% and Fatah by 1%. These encouragement effects are even stronger when focusing on more recent observations after the Oslo era.

Focusing on the effects of Fatah's competitive incentives, we find evidence of outbidding's unexpected discouragement effect: when Fatah has greater incentives to compete, violence by both groups decreases. We estimate that a 1% increase in Fatah's effectiveness results in a 1 percentage point decrease in the frequency of terrorism by Hamas and a 0.2 percentage point decrease in the frequency of terrorism by Fatah. On average, this implies both groups would decrease their violence by 2% if Fatah were to have greater incentives to compete via becoming 1% more effective at outbidding. Again, these discouragement effects are even stronger after the Oslo era.

In Appendix A, we repeat the same exercise for the value of support, β_i, and the costs of attacking, κ_i—see Figures A.3 and A.4, respectively. Shifts in β_i can be interpreted as policies that make the actors more or less dependent or the population for support. Increased or improved democracy in Palestinian territories, for example, may raise β_i for both sides, while increased foreign aid may decrease β_i for the recipient as it makes them more dependent on outside support and less on the public. Likewise, increased foreign aid to Israel or technological shifts such as the Iron Dome may change the costs of terrorism κ_i. The main takeaways are similar: when Hamas becomes more competitive, both sides attack more frequently (as expected by the outbidding literature), but when Fatah becomes more competitive, both sides tend to attack less frequently (in contrast to expectations in the outbidding literature).

These discouragement effects arise from asymmetric competition. Fatah is a relatively advantaged player due to its effectiveness at using terrorism to increase public support, that is, $|\gamma_{F,1}|$ is substantially larger than $|\gamma_{H,1}|$. When Fatah's incentive to compete increase, it more readily absorbs the up-front costs of terrorist attacks to increase public opinion levels in the future. This affects Hamas's equilibrium strategy. When Fatah becomes more aggressive, Hamas generally attacks less as it cannot efficiently compete against the more aggressive and more capable Fatah. In equilibrium, this creates a feedback loop where Fatah uses less violence as Hamas becomes more nonviolent. Thus, stronger incentives to compete against a rival for one group can deter terrorism from all groups.

8 Discussion

As shown in the above counterfactuals, the relationship between intergroup competition and terrorism is not as clear as the previous literature suggests. Whereas most studies looking for empirical evidence of outbidding focus on uncovering an encouragement effect in which enhanced competition leads to more violence, we find discouragement effects can also exist in a theory of outbidding where competition depresses violence. The key difference is the structural approach: we write down a model of outbidding, fit the model to observed data in the Fatah-Hamas rivalry, and then quantify the effects of competition on violence in the fitted model.

These heterogeneous effects matter for both researchers and policymakers. To see this, consider the effect of changes in the costs of terrorism, κ_i. For example, Israeli officials may want to pursue policies that make it harder for these groups to acquire arms or raise funds, e.g., barriers, trade restrictions, or violent reprisals. Likewise, scholars would like to know how well a reduced-form study captures the relationship between competitive incentives (κ_i in this example) and the probability of violence. Increasing the costs of terrorism will decrease both groups' incentives to compete, and if we focus on just the encouragement effect, then we may anticipate that these changes should lead to less violence overall. However, with heterogeneous effects, these implications are less clear.

Table 5: Average attack probabilities as costs κ_H and κ_F change.

		Pr(Hamas attacks)	Pr(Fatah attacks)	Pr(Either attack)
Baseline		0.37	0.11	0.43
Increase costs for	Hamas	0.33	0.10	0.40
	Fatah	0.46	0.10	0.51
	Both	0.36	0.10	0.43
Decrease costs for	Hamas	0.44	0.12	0.50
	Fatah	0.35	0.11	0.42
	Both	0.38	0.11	0.44

We illustrate the implications of changes in attack costs in Table 5. These counterfactuals follow the same procedure used to create Figure 6, only here we adjust κ_i by ± 0.13 for each actor individually (reflecting policy responses targeting a single group) and then for both actors (reflecting indiscriminate policy responses that affect both groups). This number translates into a roughly 5% and 15% change in the costs of terrorism for Fatah and Hamas, respectively. The values in this table report the attack probabilities for Hamas, Fatah, and the probability of observing an attack by either group, averaged over all values of state variable.[34]

The first thing to note is increasing only Hamas's attack costs has the desired effect; Hamas commits fewer attacks on average and the overall rate of violence drops, i.e., higher costs for Hamas discourage violence. The opposite effect appears when increasing only Fatah's attack costs, i.e., higher costs for Fatah encourage violence. When only Fatah has higher attack costs, Hamas sees an opening in competition, and their average attack probability increases dramatically with the average probability of overall violence in any given month rising above 0.5. But what happens when both groups are targeted and their costs raise by the same amount? In this counterfactual, we see that encouragement and discouragement effects cancel out, and the overall attack probability is unchanged. We also see a similar washing out when looking at the effect of decreasing costs to terrorism for both groups.

The implications for policy and research are clear. Simple tactics like trying to reduce terrorism by raising its cost may not have the desired effect in a competitive environment.

[34] The takeaways are identical if we weight these averages by the number of times each state is observed in the sample.

Indeed, indiscriminate tactics that target all groups can even lead to no changes in the average probability of terrorism as the competitive incentives cancel each other out. Boosting Fatah and targeting Hamas appears to provide the most effective path away from terrorism in this conflict.

For researchers who study outbidding, this heterogeneity is concerning. Traditionally, outbidding scholars test their theories by regressing terrorist attacks on proxies for incentives to compete. What Table 5 makes clear, however, is that *even when these incentives have strong marginal effects when focusing on a single actor*, the overall effect can be a wash when incentives are changing for multiple actors within a conflict. Such an scenario can arise even when the competitive incentives are changing by the same amount in the same direction for all actors. As such, standard approaches based on correlations between violence and proxies for competition cannot falsify the outbidding hypothesis. In this case, researchers regressing violence on the costs of outbidding may mistakenly conclude that outbidding is not a factor between these groups because when both actors become more or less competitive (via changes in κ_i) the overall probability of attacks is unchanged.

In contrast, the structural approach provides a method for directly modeling competitive incentives, estimating the treatment effects of changing these incentives, and quantifying how well outbidding explains the data relative to over theories of terrorism. In this paper, we do this with outbidding theory and uncover heterogeneous effects without relying on the need for commonly used, but untestable, proxies for competition, e.g., the overall number of terrorist attacks within a given time period. We are also able to assess the model for face-validity and then explicitly consider the fit of the model both in terms of how well it explains violence on its own and in comparison to two alternative models that do not contain outbidding.

Our specific structural model can be straightforwardly applied to other cases of intergroup competition. For example, competition among republican groups in Northern Ireland, leftist groups in Colombia, or Tamil groups in Sri Lanka are natural places to study outbidding. The main limitation to studying alternative conflicts is the need for public support data, but as intrastate conflict data becomes more fine-grained, we anticipate more applications outside the specific Hamas-Fatah rivalry. In addition, the model can be applied outside of intrastate conflict because arms races and great power competition for proteges can be modeled as dynamic contests.

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