Endogenous Issue Salience in an Ownership Model of Elections*

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Abstract

We analyze a model of electoral competition based on the issue-ownership theory of campaigns. In the model, parties invest resources to manipulate the salience of various issues, and the salience of an issue is the probability a voter casts her ballot according to her party preferences on that issue. Parties use campaigns to prime voters to think about different issues. Our results uncover Riker’s “dominance principle” and suggest that parties will generally campaign on one issue. The two-dimensional version of the model demonstrates that parties talk past each other and indicates that competition will be most fierce when parties are similarly effective campaigners and the issues are not naturally salient. With more than two parties, there is a potential for free-riding on the campaigns of parties who are the most effective.

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1 Introduction

The skills of political leaders who must maneuver for public support in a democracy consist partly in knowing what issue dimensions are salient to the electorate or can be made salient by suitable propaganda.

Donald E. Stokes, 1963

One prominent explanation of electoral competition revolves around the idea that parties mobilize voters by emphasizing policy issues rather than by adopting policy platforms on the same issues (e.g., Budge and Farlie 1983a,b; Budge 1987; Budge et al. 2001; Petrocik 1996). Central to this view are two theories of issue-based vote choice: the issue ownership theory and the issue salience theory. According to the first, voters perceive one party to be the most competent to handle certain policy areas. As Budge and Farlie put it, “electors make a clear connection between a certain party and good government performance on a particular issue (…) The result is that parties are widely perceived as ‘owning’ certain issue types” (1983a, 25). Issue salience refers to the perceived importance of the dimensions that define the political space. Proponents of this theory argue that “the issues which influence political judgments are those an individual feels are important” (Rabinowitz, Prothro and Jacoby 1982, 42). An immediate consequence of this observation in the realm of voting behavior is that, “while campaigns are indeed multidimensional affairs (…) this does not mean that each voter takes all, or even many, of these dimensions into account when voting” (Hammond and Humes 1993, 142). Taken together, these theories imply that voters cast their ballots for the owner of the issue that is more salient at the time of the election; “once electors decide which issue is salient, the question of which party to support generally follows automatically” (Budge and Farlie 1983b, 271).

Although issue ownership is usually conceptualized as fixed1 or, in Petrocik’s words, as a “critical constant” (1996, 826), issue salience is thought to be susceptible to strategic manipulation (e.g., Stokes 1963; Robertson 1976; Budge and Farlie 1983a; Hammond and Humes 1993; Petrocik 1996). In this paper we examine the question of how parties decide what issues to emphasize, or make more salient, during an electoral campaign. To do so, we present a model of electoral competition in which parties manipulate the relative salience of issues by investing campaign resources. In addition, our setup allows us to address other relevant puzzles related to this view of party competition: What is the optimal number of issues a party should emphasize? How much will parties invest on each issues? When will political parties emphasize the same issues? And how does a party’s ability to increase the salience of an issue affect the strategies of other parties?

The model has three key features. First, following the extant substantive literature, we assume that voters have preferences over political parties for each policy issue and that they vote for the party that owns the issue they perceive as most salient. In contrast with most formal works, we do not endow voters with “spatial” preferences, such as Euclidean or quadratic preferences. Instead, we assume voters’ choices to be guided by the “behavioral rule” of voting for the party they think is more

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1For an exception to this approach see Meguid (2007).
competent to handle what they consider to be the most important issue at the time of the election. We believe that this assumption on voters’ behavior is more in line with the substantive literature on issue ownership, which does not include party positioning in its formulation (e.g. Budge and Farlie 1983b). To illustrate what this assumption implies, consider a voter who thinks that the Democrats are best able to cope with unemployment and that the Republicans are the most competent to deal with national defense; if national defense is the most salient issue for this voter, then she would vote Republican. This description of voters’ behavior is consistent with the observation that “issue ownership should only affect the decision of those voters who think that the issue in question is important” (Bélanger and Meguid 2008, 479). More importantly, we let perceptions about both the ownership and the salience of issues vary across the electorate. That is, we allow voters to disagree about which party is best able to handle each policy dimension, and about what issue is the most salient at the moment they cast their ballots. Again, this heterogeneity is compatible with the empirical literature, which has shown that “there is rarely a clear-cut consensus about the ownership of issues” (Bélanger and Meguid 2008, 482).

Second, we follow the extant literature in that we treat ownership as given and salience as endogenous to electoral competition. This means that the parties’ campaign efforts will affect the voters’ perceptions about what issues are important but not their beliefs regarding what party is more competent to handle them. Scholars have identified different reasons for the stability of issue ownership, such as the parties’ records in office (e.g. Budge and Farlie 1983b) and the structural attributes of the constituencies they seek to attract (e.g. Petrocik 1996). By contrast, saliency is a short-term phenomenon; the salience literature assume that voters can be persuaded that any issue is salient. As Petrocik explains, “issue concerns change with existing national conditions, but non-ideological, instrumental, and sociotropic voters are prepared to believe that almost any problem is important and they are susceptible to priming and framing efforts by the candidates” (1996, 830). Although the focus of the model is on how parties can change the relative salience of issues, we also deal with the fact that elections do not take place in a vacuum. Thus, we allow for issues to have an exogenous or “natural” salience, which is intended to capture two realistic features of issue competition. The first one has to do with the fact that certain issues – the economy, for example – are, in general, more salient than others. The second one is the possibility that the electorate might shift its attention to certain issues for reasons beyond the control of parties; for instance, the salience of national security issues might increase after a terrorist attack.

Third and final, we examine a situation in which parties are endowed with resources, which they can use to manipulate the relative saliency of issues. In the model, the salience of an issue corresponds to the probability that a voter casts her ballot according to her preferences on that issue, and its key feature is that increasing the salience of one issue decreases the salience of other issues. In this context, parties essentially use campaigns to prime voters to care about different issues. We allow campaign expenditures to have heterogeneous effects across both parties and issues. This means two things. On the one hand, not all parties are equally capable of altering the salience of the same issue. To illustrate this point, we could think that the Green Party is more effective at increasing the salience of environmental issues than the Conservative Party. On the other hand,
the ability of a party to change the salience of policy dimensions varies across issues. That is, a party can simultaneously be very successful at drawing attention to one issue but almost incapable of shifting the salience of others – such is the case of niche parties, which are by definition single-issue parties (Meguid 2007).

In the model, when parties decide how to spend their campaign resources, they care about two important links between themselves and the possible issues: ownership and campaign effectiveness. In this framework, issue ownership refers to a voter’s belief concerning which party possesses the best policy reputation on the issue or which party best solves problems associated with the issue. Effectiveness, in contrast, refers to the degree to which parties actually influence voters’ perceptions about the relative importance of issues. Parties who are not effective on an issue will need to spend more resources to convince voters that the issue is important than parties who are effective. Our main results are driven by the interaction between ownership and effectiveness.

Our results offer confidence that the model captures important aspects of substantive theories of issue ownership and, at the same time, provide avenues for future empirical and theoretical research. In the more general case we consider, with an arbitrary number of parties competing along an arbitrary number of dimensions, we find that a party will never campaign on its weakest issue and will almost always invest its resources on one issue. In our view, when every party is generically investing in at least one issue, then the number of parties can serve as an upper bound on the number of issues that are emphasized during a campaign.

For the case of two parties competing over two issues, we find that parties “talk past each other,” as they never invest on the same issue. Even though this finding is consistent with foundational work on this area (Budge and Farlie 1983b; Petrocik 1996), the model uncovers some insights on how parties decide what dimensions to emphasize. Whereas the conventional rationale for this result is that a party’s calculus involves considering which party has the upper hand on each issue, our model suggests – and this applies to the more general case too – that each party will make this decision by considering what issue is more advantageous relative to other issues. To illustrate this difference, consider the extreme case where voters are more likely to think the same single party owns every issue. While a traditional interpretation of the theory would suggest that other parties should remain silent, our model suggests that, even in this case, all parties have at least one issue that is more profitable to emphasize relative to other issues.

In addition, we identify an inverse-U relationship between a party’s effectiveness at increasing the salience of the issue and its campaign spending. That is, a party with either small or high levels of effectiveness at increasing the salience of an issue will invest fewer resources than if it had middle effectiveness levels. Non-linearities also arise when we consider the effects of a party’s efforts on other parties’ campaign strategies. In two party competition, where both parties emphasize different issues, we find that a party’s campaign spending in its favorable issue has an inverse-U with both the competing party’s effectiveness to increase the salience of the remaining issue and with the other party’s cost of campaigning. In other words, electoral competition is most fierce
when parties have similar, but not identical, campaign costs and effectiveness.

Finally, we consider an example with three parties. We characterize an equilibrium in which two parties invest in the same issue, which highlights potential free-riding problems in electoral campaigns. The result indicates that when distinct parties both campaign on an issue, one party exhausts all of its resources on the issue, and another party still finds it beneficial to increase the issue’s salience after this initial investment. Substantively, this result provides another reason why parties campaign on the same issue besides attempting to hurt the vote share of a rival (Meguid 2007) or to steal ownership (Holian 2004; Sides 2006).

This paper is organized as follows. In the next section, we briefly review the literature on issue ownership as well as the formal works that are more closely related to ours. In Sections 3 and 4, we present the model and describe general results of substantive interest, respectively. Section 5 then presents the main results for the two–party, two–issue case, and Section 6 shows an example of three–party competition that illustrates the potential for free riding in electoral campaigns. Section 7 concludes.

2 Literature Review

The notion that the emphasis of policy issues is at least as important as the adoption of policy stances on particular issues for electoral competition can be traced to Stokes’ (1963) famous critique of the Downsian (1957) framework. In it, Stokes makes a distinction between what he calls position issues, which “involve advocacy of government actions from a set of alternatives over which a distribution of voter preferences is defined,” and what he calls valence issues, which “merely involve the linking of the parties with some condition that is positively or negatively valued by the electorate” (1963, 373). One of his main criticisms is that the spatial model of party competition cannot accommodate valence issues because, when dealing with this kind of policy dimension, party strategy is essentially about choosing what dimensions of political evaluation should be emphasized; as he explains, “electoral changes can result from changes in the coordinate system of the space rather than changes in the distribution of parties and voters” (1963, 372).

In the same vein, Robertson (1976) introduces a “problem-solving” approach to electoral competition. In his view, voters are often incapable of technical appraisal of policy itself and instead must rely on their evaluations about the competence of political parties, seen as alternative governments, to solve the problems that require government action. This assumption on voters’ knowledge, he argues, implies that party competition consists of identifying the problems that need to be solved and offering solutions for them; voters “cannot set the problems which are to be dealt with, but must choose one alternative that represents both a list of problems and suggested solutions” (1976, 12). Furthermore, Robertson argues that over time parties will develop reputations regarding their competence to handle certain policy dimensions so that different issues will become advantageous for certain parties.

Probably the most comprehensive formulation of this theory is that of Budge and Farlie (1983a; 1983b, see also Budge 1987; 2001). According to these authors, parties are endowed with issue reputations; that is, voters distinguish parties in terms of their ability and competence to bring
about desirable outcomes in specific policy areas. Therefore, political parties compete by selectively emphasizing the issues that work for them and by ignoring the ones that are more advantageous for others. As they put it, “To win votes therefore strategists do not argue much about policy positions, which are taken as read, but emphasize the importance of those issues where the party is ideologically committed and hence most trusted by electors” (1983a, 82). Petrocik’s (1996) theory further elaborates on the sources of these differentiated issue reputations, or issue ownership, and argues that these are a function of both the record of the incumbent and the constituencies of the parties. As he explains, “party constituency ownership of an issue is much more long–term because its foundation is (1) the relatively stable, but different social bases, that distinguish party constituencies (…) and (2) the link between political conflict and social structure” (1996, 827). Since parties have distinct bases of support, there is a strong connection between a party’s issue agenda and the social characteristics of its constituency, and these factors reinforce each other election after election.

By and large, these works have concluded that, in the context of electoral campaigns, parties will try to increase the salience of the issues they own and ignore the ones that are owned by other parties. This proposition comes under different names. Riker’s (1996) “dominance principle” states that “when one side has an advantage on an issue, the other side ignores it” (106). Budge and Farlie (1983a) refer to this as the “selective emphasis” of issues, so that “rather than promoting an educational dialogue, parties talk past each other” (24). Similarly, according to Petrocik’s “issue ownership” theory the goal of political parties in electoral campaigns is “to achieve a strategic advantage by making problems which reflect owned issues (…) the criteria by which voters make their choice” (828).

Although this literature emerged and developed explicitly as an alternative to Downs’ (1957) spatial model, it is surprising to see that most of the formal work on this area has proceeded by merging the two approaches into one (e.g. Amorós and Puy 2010, 2013; Aragonès, Castanheira and Giani 2012; Hammond and Humes 1993; Rozenas 2009; Simon 2002). In Simon (2002), Amorós and Puy (2010, 2013), and Rozenas (2009), it is assumed that voters’ utility functions take the form of quadratic distance between the voter’s ideal point and the candidates’ platform on the issue. A common problem with this setup is the difficulty in determining the existence of pure strategy equilibria. In Aragonès, Castanheira and Giani (2012), the utilities are linear and strictly increasing in the candidates’ positions on the issue because the policy platforms are public goods and the position reflects the quality of party’s policy addressing such issue. Roth (2011) investigates a decision-theoretic model and discusses the possibility of free-riding in this context.

Our model is also related to a set of works that consider candidates with different productivities in distinct policy areas who compete by choosing the amount of resources to allocate on each dimension (e.g. Krasa and Polborn 2010; Hummel 2012; Pollack 2011), and, more generally, to models in which players compete by spending resources on a number of simultaneous contests (e.g. Cox and McCubbins 1986; Kvasov 2007; Kovenock and Roberson 2012). The main difference between our model and the ones commonly found in this latter literature is our interpretation of an issue’s salience. In the model below, salience is not the probability that a party wins or loses a specific issue but rather the probability that a voter determines her vote choice according to her preferences on that issue. Because of this, increasing the salience of one issue decreases the salience of all other
issues, which leads to richer comparative statics and potential difficulties in applying off-the-shelf existence arguments.

3 The Model

In this section, we present a model in which parties or candidates invest campaign resources across issues to manipulate their salience. In the model, the salience of an issue corresponds to the probability that a voter votes according to her preferences on that issue, and its key feature is that increasing one issue’s salience decreases the salience of all other issues. This framework incorporates the ideas that every voter can be convinced that every issue is important and a voter’s political preference on an issue matters in her vote choice only when the issue is salient or important to her.

There are $J$ parties competing to win the votes of a unit mass of voters along $K$ issues. Hereafter, we use $j = 1, ..., J$ to index an arbitrary party and $k = 1, ..., K$ to index the issues. Parties simultaneously spend resources to influence the salience of each issue, which then influences the parties’ vote shares; that is, candidates use campaigns to prime voters to care about certain issues (Jacobs and Shapiro 1994; Druckman 2004; Bartels 2006). Party $j$ possesses $R_j > 0$ resources, which can be allocated across the $K$ issues, and $j$’s action set, denoted $\Lambda^j$, consists of all feasible allocations of $j$’s resources. That is,

$$\Lambda^j = \left\{ \lambda^j \in [0, R^j]^K \mid \sum_k \lambda_k \leq R^j \right\}$$

Let $\Lambda = \Lambda^1 \times \cdots \times \Lambda^J$ denote the set of possible resource allocations for all parties. For any resource allocation $\lambda$ the resulting salience of dimension $k$ is given by the function $\bar{s}_k : \Lambda \times [0, 1]$ where

$$\bar{s}_k(\lambda) = \frac{\sigma_k + \sum_j \beta^j_k \lambda^j_k}{\sum_{k'} \sigma_{k'} + \sum_j \beta^j_{k'} \lambda^j_{k'}}$$

$\beta^j_k > 0$ is the relative effectiveness of party $j$’s investment, and $\sigma_k > 0$ is the natural salience of issue $k$ if no campaign resources are spent. The relative effectiveness parameters capture situations where a party may be more or less effective at increasing the salience of certain issues. For example, a Green Party campaign highlighting the dangers of nuclear energy increases the salience of environmental issues more than a Green campaign about public safety increases the salience of law and order issues. In addition, the effectiveness parameters also capture inter-party differences where the conservatives more effectively increase the salience of foreign policy than communist parties. In other words, these effectiveness parameters capture a party’s ability to convince voters an issue is important. In contrast, the natural salience parameters $\sigma_k$ model situations in which certain issues are inherently more important to voters. For example, on average, voters find economic performance more important in their vote choice than farm subsidies.

In words, $\bar{s}_k$ maps campaign spending into the salience of dimension $k$. We use $\bar{s}(\lambda) = (\bar{s}_1(\lambda), ..., \bar{s}_K(\lambda))$ to denote the product of the salience functions. While the analysis to follow assumes the specific functional form in Eq. 1, it has a very intuitive interpretation. The salience of
an issue, or the probability that a voter thinks the issue is important, is the amount of resources devoted to the issue divided by the total of all the resources spent in the campaign. In addition, \( s_k \) captures three important features of issue salience which are common in the literature (Amorós and Puy 2010, 2013; Aragonès, Castanheira and Giani 2012; Rozenas 2009).\(^3\) First, the salience of issue \( k \) is strictly increasing in each party’s investment into the issue (\( \lambda_k^j \)) and the issue’s underlying population salience (\( \sigma_k \)). In other words, campaigning on an issue only makes voters more likely to vote according to that issue. Second, \( s_k \) imposes a decreasing marginal rate of return on investment in the sense that a party becomes less effective at manipulating the salience of any issue when considerable resources have already been spent. That is, a party with a campaign budget of $10,000 more effectively manipulates the salience of various issues when the other parties are spending $1,000 rather than $1 million. Third, and most importantly, issue \( k \)’s salience is strictly decreasing in the parties’ investment in other issues and the natural salience of other issues. Thus, when all parties campaign on economic performance this drowns out the importance of other issues in the election.

Party \( j \) seeks to maximize its expected vote share \( V^j \). Here we assume that voters (who need not be modelled) vote according to their preferences on the issues they believe is most important, and this most important issue is determined stochastically from party campaigns. To account for this, when issue \( k \) is completely salient under profile \( \lambda \), i.e. \( s_k(\lambda) = 1 \), all voters vote according to their preferences on issue \( k \), and party \( j \)’s vote share is \( v^j_k \in [0, 1] \). We label \( v^j_k \) as party \( j \)’s expected vote share on issue \( k \) and assume that for all issues \( k \), \( \sum_j v^j_k = 1 \).\(^4\) When no single-issue is perfectly salient, a party \( j \)’s expected vote share is a weighted sum of \( v^j_k \). Specifically, for a profile of investments \( \lambda \), party \( j \)’s vote share can be written as

\[
V^j(\lambda) = \sum_k \hat{s}_k(\lambda) v^j_k.
\]

Thus, voters are not optimizing a spatial utility function when casting their ballots and only the issue they care about influences their vote choice. Notice that the issue on which they base their vote is a random variable, and in this sense, our model is behavioral. Voters potentially care about many issues, but when they walk into a ballot box, they choose an issue on which to vote according to some probability distribution.

Party payoffs are their vote share minus their campaign efforts. Formally, the utility function of party \( j \) is \( u^j : \Lambda \to \mathbb{R} \) where

\[
u^j(\lambda) = V^j(\lambda) - \delta^j \sum_k \lambda_k^j,
\]

\(^3\)In particular, the first and third assumptions are common to Amorós and Puy (2010, 2013); Aragonès, Castanheira and Giani (2012); and Rozenas (2009), and the second assumption is satisfied by many of the examples in Amorós and Puy (2010).

\(^4\)While there is a significant possibility that a party’s effectiveness at campaigning on an issue, \( \beta_k^j \) and its expected vote share on the same issue \( v^j_k \) are positively correlated, we need not place further assumptions on the relationship between the two parameters. Furthermore, in dominant two-party systems, e.g. the United States, parties such as the Green Party may face an added difficulty in influencing the thought-process of voters’ even on environmental issues due to their non-mainstream status.
for all allocations $\lambda \in \Lambda$. The parameter $\delta^j \geq 0$ captures the cost of campaign effort or the marginal rate of substitution between resources and vote share for party $j$. When $\delta^j = 0$, the party has a budget $R^j$ and seeks to spend all its money. When $\delta^j > 0$, the party values its resources, which could happen if campaign resources carry over to the next election or if resources also include volunteers who may become disillusioned after working for too many hours.

In the game where parties are choosing $\lambda^j$ simultaneously, our equilibrium concept is Nash, where a strategy for party $j$ is an allocation of resources across issues.

### 4 Remarks

Two remarks are in order before proceeding to the analysis of the game. First, we offer two interpretations about the parties’ vote shares across issues, one based on issue ownership and another based on the Downsian model. Second, we briefly discuss previous models of parties manipulating issue salience and how these models differ from ours and the substantive literature detailed above. According to the vote ownership literature, parties own certain issues, and once voters decide what issue is important, they vote for the party that owns the issue (Budge and Farlie 1983b; Petrocik 1996). Although some approaches model issue ownership as fluid (Aragonés, Castanheira and Giani 2012; Meguid 2007), Petrocik (1996, p. 826) labels issue owners as the “critical constants” in an electoral system originating from the historical performance and support of each party (see also Bowler 1990). Thus, we can interpret $v^j_k$ as the probability that a random voter believes party $j$ owns issue $k$ (or solves problem $k$ better than the other parties). Therefore, vote choice need not reflect strategic voting or minimizing the distance between a voter’s ideal policy and a candidate’s because these beliefs are stochastic and subject to manipulation. Furthermore, in the pure ownership model, we can assume that for each issue $k$, there exists party $j$ such that $v^j_k = 1$ to capture the possibility that every issue has one unique owner and when a voter cares about issue $k$ she votes for its owner. However, empirically, studies find that voters associate multiple owners with a single issue (Budge and Farlie 1983b; Petrocik 1996; Bélanger and Meguid 2008), and this holds even in two party systems. Therefore, we do not restrict the general setup to a pure ownership model but acknowledge that it is an important specification of the expected issue vote shares.

Nonetheless, the model presented in the last section does not rule out a more Downsian interpretation when voters’ spatial preferences over parties along one dimension do not depend on the parties’ policy positions along all other dimensions. In this case, the voters have separable policy preferences, and the parameter $v^j_k$ denotes the percentage of the electorate that has an ideal point on dimension $k$ closest to party $j$. In other words, voters have circular indifference curves or elliptical-shaped ones whose axes correspond to the Cartesian axes, and when a voter cares about an issue, she votes according to her party preferences on that issue, which does not depend on the party’s other issue positions, by assumption.\(^5\) To better see how our framework incorporates this type of electoral competition, suppose there are two dimensions and three parties. The parties have policy platforms $\hat{x}^j$ and assume the unit mass of voters – indexed by $i$ – have ideal points $\hat{x}^i$ distributed over $[0, 1] \times [0, 1]$ according to the cumulative distribution function $F(x_1, x_2)$. When voter $i$ believes

\(^5\)For a more in-depth discussion of separable preferences see Austen-Smith and Banks (2005, p. 62).
Figure 1: A Downsian interpretation of the model. Here voters have ideal points \( \hat{x}^i \) distributed over \([0, 1] \times [0, 1]\) according to distribution \( F(x_1, x_2) \), with corresponding marginal distributions \( F_1(x_1) \) and \( F_2(x_2) \). When voters who care about issue \( k \) minimize the distance between their ideal policy on issue \( k \) and the policy platform of their vote choice, we can compute a party expected vote share on issue \( k \) using the marginal distributions. For example, Party 3’s vote share on issue 2 is \( v^3_2 = 1 - F_2 \left( \frac{\hat{x}^2 + \hat{x}^3}{2} \right) \), and Party 1’s vote share on issue 1 is \( v^1_1 = F_1 \left( \frac{\hat{x}^1 + \hat{x}^3}{2} \right) \).

Dimension \( k = 1, 2 \) is important, she chooses the party \( j \) that minimizes the distance \(|\hat{x}^j_k - \hat{x}^i_k|\). For example, the policy platforms could look like those in Figure 1. Then Party 1’s expected vote share on issue 1 can be expressed as \( v^1_1 = F_1 \left( \frac{\hat{x}^1 + \hat{x}^2}{2} \right) \), where \( F_1(x_1) \) is the marginal distribution of \( F \) with respect to \( x_1 \). Likewise, Party 2’s expected vote share on issue 1 is \( v^2_1 = F_1 \left( \frac{\hat{x}^2 + \hat{x}^1}{2} \right) - F_1 \left( \frac{\hat{x}^1 + \hat{x}^2}{2} \right) \), and Party 3’s is \( v^3_1 = 1 - F_1 \left( \frac{\hat{x}^1 + \hat{x}^2}{2} \right) \). We compute the expected vote shares over the second issue in a similar manner, but now the ordering of the party changes and we use the marginal distribution over \( x_2 \), \( F_2 \). While this Downsian interpretation still depends on our critical behavioral assumption that voters care only about one issue and this issue is determined stochastically according to the parties’ campaigns and natural salience, it illustrates that the model of campaigning presented in the previous section addresses these two theories of voting behavior.

Second, previous models of issue salience also assume that voters have separable preferences across issues. In general, they take the following form. A voter \( i \)'s utility function can be written as

\[
u^i(j) = \sum_k \alpha_k u^i_k(j),\]
where $u_i^k$ is voter $i$’s utility function for issue $k$ and $\alpha_k$ are the endogenous salience parameters. In Amorós and Puy (2010, 2013) and Rozenas (2009), these component utility functions take the form of quadratic distance between the voter’s ideal point and candidate $j$’s platform on issue $k$. In Aragonés, Castanheira and Giani (2012), the competent utilities are linear and strictly increasing in $\hat{x}_k^j$ because the policy platforms are public goods in the sense that $\hat{x}_k^j$ is the quality of party $j$’s policy addressing issue $k$. Unlike our model, these previous ones assume voters care about more than one issue at time when deciding their vote choice. However, this setup comes with two potential drawbacks. Theoretically, this makes the calculations of candidates more complicated, which results in an absence of a pure strategy equilibrium (Amorós and Puy 2010, 2013; Rozenas 2009) even with two parties and two issues or the needed assumptions of candidate symmetry and preference homogeneity of voters over party platforms (Aragonès, Castanheira and Giani 2012). Substantively, it does not capture the interaction between a voter’s preferences over issues and her perception of whether the issue is important. Thus, the setup may not model the types of voters prevalent in the issue-ownership theories.

5 Dominated Strategies and Equilibrium Existence

This section discusses the possibilities of strictly dominated and dominant strategies and the necessary conditions for a party to invest in multiple dimensions. We then use these results to identify sufficient conditions for the existence of a Nash Equilibrium.

Intuitively, party $j$’s vote share is a weighted average of the party’s expected vote share across issues, where the weight or the importance of an issue is strictly increasing in its dedicated resources. When campaign resources are valuable, i.e., $\delta^j > 0$, or when the smallest expected vote share is different than the largest, a party would never want to invest in the issue with the smallest vote share. Doing so would only decrease its average vote share, $V^j$, with some potential cost. The next lemma states this formally.

**Lemma 1** Suppose there exists party $j$ and dimension $k$ such that $v_k^j \leq v_k^{j'}$ for all issues $k'$. If $\delta^j > 0$ or if $v_k^j \leq v_k^{j'}$ holds with a strict inequality for one $k' \neq k$, then any strategy $\lambda_k^j$ such that $\lambda_k^j > 0$ is strictly dominated strategy.

**Proof.** The proof follows directly from the derivative of $u^j$ with respect to $\lambda_k^j$, denoted $D_ku^j$. Specifically, we have

$$D_ku^j = \frac{\sum_{k'}(v_k^j - v_k^{j'}) \left[ \sigma_k + \sum_{j'} \beta_k^{j'} \lambda_k^{j'} \right]}{\left( \sum_{k'} \left[ \sigma_{k'} + \sum_j \beta_{k'}^{j'} \lambda_{k'}^{j'} \right] \right)^2} - \delta^j$$

$$= \frac{v_k^j - V^j(\lambda)}{\left( \sum_{k'} \left[ \sigma_{k'} + \sum_j \beta_{k'}^{j'} \lambda_{k'}^{j'} \right] \right)} - \delta^j. \quad (3)$$

Note that $v_k^j - v_k^{j'} \leq 0$ for all dimensions $k'$ under the conditions of the proposition. Then $\delta^j > 0$ or $v_k^j < v_k^{j'}$ for some dimension $k'$ ensure that $D_ku^j$ is strictly less than zero for all strategy profiles.
\( \lambda \). Hence, any strategy \( \lambda^j \) such that \( \lambda^j_k > 0 \) will be strictly dominated by the strategy \( \mu^j \in \Lambda^j \) such that \( \mu^j_k = 0 \) and \( \mu^j_{k'} = \lambda^j_{k'} \) for all \( k \neq k' \).

We label the issue that gives a party its lowest (highest) expected vote share as its dominated (dominant) issue. Equation 3 also reveals the utility calculations behind the parties’ campaign strategies. Party \( j \)'s marginal benefit in emphasizing issue \( k \) increases as the difference between \( j \)'s expected vote share on issue \( k \) and some other issue \( k' \) increases. Thus parties look for relative advantages rather than absolute advantages. This is a departure from previous expositions of issue-ownership theories of party competition in which parties emphasize issues which they own or will win on if they can convince the electorate the issue is important. It is therefore possible that a party campaigns on an issue even if the expected vote on the issue is very small. In this case, its expected vote share on other issues is even smaller. In addition, the natural salience of issue \( k \), \( \sigma_k \), decreases the marginal effect in Eq. 3. Thus, as an issue becomes more naturally important to voters, candidates will spend less time and energy convincing voters that the issue is important.

Unfortunately, a clean characterization of strictly dominant strategies and issues does not exist because a party’s benefit from investing in an issue depends on the amount of resources already devoted to the issue, its effectiveness of campaigning on the issue, and the degree to which it owns the issue, i.e., the size of \( v^j_k \). To see this, note that even if party \( j \) completely owns issue \( k \), i.e., \( v^j_k = 1 \), the party may not campaign on this issue if \( \delta^j \) or \( \sigma_k \) are sufficiently large or if it is not very effective at campaigning on this issue. Nonetheless, once we fix a profile actions of all other parties except one, the remaining party will almost always best respond by investing either in one issue or no issues. Parties engage in single-issue campaigning because, if they campaigned on two issues, the marginal effect of these two investments on their vote shares must be equal. If not, then parties could transfer resources from one issue to the other, thereby increasing their vote share, which means the initial allocation of resources was not a best response. The requirement that the marginal effects of two issues are equal establishes a knife-edge characterization which must hold when a party invests in two-dimensions at the same time. The next lemma states this result formally.

**Lemma 2** Consider some profile \( \lambda \). Then \( D_k w^j(\lambda) \geq D_l w^j(\lambda) \) if and only if

\[
\beta^j_k (v^j_k - V^j(\lambda)) \geq \beta^j_l (v^j_l - V^j(\lambda)).
\]

**Proof.** This follows from the derivation of the \( D_k w^j \) in Lemma 1. \( \square \)

Thus, in equilibrium, if a party invests in two issues, then the weak inequality in the above Lemma holds with equality. In other words, a party almost always invests in one issue when best responding to the campaign investments of other parties, and parties develop a unified or coherent campaign message. This result then suggests an important relationship between the number of parties and the number of issues in a campaign. When every party is generically investing in at most one issue, the number of parties can serve as an upper bound on the number of issues emphasized during a campaign. The issue emphasized by a party, however, is ambiguous because one issue may give the party its highest vote share and another may be the one on which the party most effectively campaigns. For example, the inequality in Lemma 2 illustrates that when a party’s expected vote
share is relatively large ($V^j(\lambda) \approx v^j_k$), it values issues on which they are very effective. When a party’s expected vote share is relatively small ($V^j(\lambda) \approx 0$), it values issues which they are more likely to own. Taken together, these two preliminary lemmas offer some confidence that the model captures an important aspect of previous substantive theories and provide some tractability in the subsequent analysis. The next proposition states two sufficient conditions or equilibrium existence in pure strategies.

**Proposition 1** A pure-strategy Nash equilibrium exists if any of the following conditions hold.

1. There are two issues, that is, $K = 2$.
2. Parties are most effective on their dominant issue, that is, for all $j$, there exists an issue $k$ such that $v^j_{k, j} \geq v^j_{k', j}$ and $\beta^j_{k, j} \geq \beta^j_{k', j}$ for all issues $k'$.

**Proof.** See Appendix.

Proof. See Appendix.

In words, the proposition says that a pure-strategy Nash equilibrium exists if there or two issues or there is sufficient correlation in parties’ issue vote shares, $v^j_{k, j}$, and their campaign effectiveness, $\beta^j_{k, j}$. We briefly note that straightforward applications of general existence results, e.g. the Debreu-Glicksberg-Fan Theorem (Fudenberg and Tirole 1991, p. 34), do not apply here because the general model does not guarantee that $u^j$ is quasi-concave in $\lambda^j$. Nonetheless, if we impose a sufficiently convex, rather than linear, cost of investment in Eq. 2, this would make the utility function concave and establish pure-strategy equilibrium existence more generally.\footnote{More specifically, we would write $u^j(\lambda) = V^j(\lambda) - c^j \left( \sum_k \lambda^j_k \right)$, where $c^j(x)$ is the cost to party $j$ of investing $x$ amount of resources in the campaign and $D_{2c}(x)$ is sufficiently large or positive.}

Since our results below do not require this convexity, there is little need to introduce this complication in the setup.

### 6 The Two Party and Two Issue Case

In this section, we characterize equilibria in the model when two parties campaign over two issues. Doing so provides insight into party competition in the issue-ownership model. We write the set of parties as $\{A, B\}$ to more easily differentiate parties from issues. Due to the model’s numerous parameters, there is an embarrassment of cases in a general equilibrium analysis. To simplify the analysis we make the following assumptions.

**Assumption 1** Define the following three assumptions.

1. Party $A$ “favors” dimension 1, that is, $v^A_1 > v^A_2$.
2. Resources are costly, that is, $\delta > 0$ for parties $j = A, B$.
3. $R^j$ is large for parties $j = A, B$.

In words, Assumption (1A) that Party $A$ receives a higher expected vote share on issue 1 than issue 2 is without loss of generality. If $v^A_1 < v^A_2$, we could always relabel the dimensions. If $v^A_1 = v^A_2$, then both parties investing no resources into either dimension is an equilibrium because $V^j(\lambda)$ is constant in the profiles $\lambda$, and campaigns do not effect vote shares. Indeed, when resources are costly (1B), then this is the unique equilibrium because parties would pay some cost without the ability to
increase (or decrease) their vote shares.\footnote{Without \eqref{eq:1b}, \( v^A_1 = v^A_2 \) implies every strategy profile is an equilibrium.} Taken together, the second and third assumptions mean that parties can always raise more campaign resources, but these resources are costly. Technically, \eqref{eq:1B} and \eqref{eq:1C} ensure that we do not have to worry about equilibria in which campaigns completely spend their resources \( R^j \).\footnote{When there are only two issues, \( u^j \) will be strictly concave in \( j \)'s non-dominated issue and have a maximizer, thereby guaranteeing that a party will never spend an infinite amount of resources.} Substantively, these assumptions encourage comparative statics on optimal campaign strategies and issue emphasis, which have remained absent in previous analyses. The next proposition presents the main result of the paper.

**Proposition 2** Assume \( \eqref{eq:1A}-\eqref{eq:1C} \) and there are two parties competing over two issues. Then the following results hold.

1. There exists a unique equilibrium.
2. In the unique equilibrium \( \bar{\lambda} \) in which both parties invest positive resources, Party A invests \( (\bar{\lambda}^A_1, 0) \), where

\[
\bar{\lambda}^A_1 = \frac{\beta^A_1 \beta^B_2 \delta^B (v^A_1 - v^A_2)}{(\beta^A_1 \delta^B + \beta^B_2 \delta^A)^2} - \frac{\sigma_1}{\beta^A_1},
\]

and Party B invests \((0, \bar{\lambda}^B_2)\) where

\[
\bar{\lambda}^B_2 = \frac{\beta^B_1 \beta^A_2 \delta^A (v^A_1 - v^A_2)}{(\beta^A_1 \delta^B + \beta^B_2 \delta^A)^2} - \frac{\sigma_2}{\beta^B_2}.
\]

**Proof.** See Appendix.

Thus, not only does an equilibrium exist, it is unique, and the two parties never invest in the same issue.\footnote{See the Appendix for details concerning other equilibria.} In other words, parties “talk past each other” in elections, and the model uncovers the “dominance principle” in which a party ignores the issue on which its competitor has an advantage. This result is similar to previous models of salience (Amorós and Puy 2010, 2013; Aragonès, Castanhêira and Giani 2012) which have found that issue-divergence rather than issue-convergence persists in equilibrium. In addition, this relates to Petrocik’s (1996) argument that candidates “argue along lines that play to the issue strength of their party, and sidestep their opponent’s issue assets” (p. 829) and “will not usually engage in text-book debates with each disputing points raised by their opponents” (p. 831).

When both parties make positive investments in their non-dominated issue, these equilibrium investments are functions of the model’s primitives. For example, the investments vary with the effectiveness parameter \( \beta^j \) and the cost parameters \( \delta^j \) because these determine the cost-effectiveness of a party’s campaign. In addition, the equilibrium investments depend on the natural salience of each issue because these determine how much parties need to invest to appreciably alter the salience of an issue. More importantly, the parties’ investments depend on the difference between the expected vote shares on each issue because when Party A campaigns on issue 1, this not only increases salience of its strongest issue, but it also reduces the salience of the weaker issues. In other words, there are two effects of campaign investments: a direct one through increasing the
salience of the campaign issue and an indirect one through decreasing the salience of the remaining
issues. This leads to a “spillover effect” where $B$’s effectiveness on the second issue determines $A$’s
equilibrium investment on the first issue. The intuition for the spillover is as follows. Party $A$’s
investment in issue 1 depends on Party $B$’s effectiveness in issue 2 because as $B$ becomes more
effective at campaigning on its strong issue, the party campaigns more. This means that $A$ needs
to campaign more on issue 1 if it wishes to highlight its strong issue to the same degree. The next
proposition more precisely states how these primitives affect equilibrium investments and highlights
this recursive nature of the two campaigns.

**Proposition 3** In the equilibrium in which both parties invest positive resources, the following
relationships hold.

1. Party $A$’s equilibrium investment in issue 1 is increasing in the difference in $A$’s vote shares
   across the two dimensions ($v_{1A} - v_{2A}$) and decreasing in $A$’s cost of effort ($\delta^A$) and issue 1’s
   natural salience ($\sigma_1$).
2. Party $A$’s equilibrium investment in issue 1 has an inverse-U (negative quadratic) relationship
   with $A$’s effectiveness at campaigning on issue 1 ($\beta_1^A$) and Party $B$’s effectiveness on issue 2
   ($\beta_2^B$) and cost of effort ($\delta^B$).
3. The comparative statics for Party $B$ are symmetric.

The proof of the proposition follows straightforwardly from the derivative of $\bar{\lambda}_1^A$ with respect to
its various parameters. In other words, increasing $v_{1A}$ or decreasing $v_{2A}$ incentivizes Party $A$ to invest
more resources in her dominant issue because, in the first case, the party highlights an increasingly
favorable issue and, in the second case, it drowns the importance of an increasingly unfavorable one.
As mentioned above, parties emphasize issues on which they have a relative advantage, and as this
relative advantage increases so do resources devoted to the issue. In addition, increasing the natural
salience of issue 1 decreases $A$’s optimal investment because voters already believe $A$’s favorable
issue is important. Finally, increasing a party’s cost to raise campaign resources decreases its overall
spending.

The quadratic relationships in Proposition 3 are slightly more complicated, but they illustrate
the recursive nature of the two campaigns in equilibrium. First, a party’s campaign spending on
its favorable issue has a inverse-U relationship with its effectiveness on the issue and the competing
party’s effectiveness on the remaining issue. For example, when $B$’s effectiveness on its favorable
issue ($\beta_2^B$) increases at small values, $B$ more easily increases the salience of her issue, which means $A$
must spend more to convince voters her issue is important. However, when ($\beta_2^B$) increases at large
values, $A$ realizes $B$ is very effective at campaigning and begins to spend less resources. In turn, this
means $B$ needs to exert less effort to convince voters. More generally, in the interior equilibrium,
$A$’s spending is maximized when $B$’s effectiveness is

$$\frac{\beta_1^A \delta^B}{\delta^A}.$$

At this value, $B$’s equilibrium spending is strictly increasing in her effectiveness.\footnote{This relationship follows from necessary and sufficient first and second order conditions on $\bar{\lambda}_1^A$ and the first}
Figure 2: A party’s level of campaign spending has an inverse-U relationship with its effectiveness on its favorable issue and the effectiveness of the other party on the competing issue.

share the same cost of raising money ($\delta^A = \delta^B$), then the campaign will be most fierce, or consume the largest amount of resources, when one party is slightly more effective than other.

Figure 2 presents a graphic representation of this logic. In this graph, the horizontal axis denotes $B$’s effectiveness on the second issue and the vertical axis the parties’ equilibrium campaigning efforts. Initially, for $\beta^B_2$ close to zero, Party $B$’s effectiveness has a positive relationship with Party $A$’s campaign spending, because as $B$ finds it easier to spend, $A$ compensates. However, when $\beta^B_2$ becomes larger, it has a negative relationship with $A$’s spending levels because $A$ cannot adequately compete against Party $B$. As $A$ campaigns less, Party $B$ no longer needs to exert much effort into the campaign.

In a similar vein, when Party $B$’s cost of campaigning increases, the party campaigns less. This means that $A$ can more easily influence the salience of its issue, but at the same time, it does not need to invest considerable campaign resources to do so. When $\delta^B$ is small, the first incentive to spend more dominates. When $\delta^A$ is large, the second incentive to spend less dominates. This results in an inverse-U relationship between Party $A$’s spending and $B$’s cost of resources similar to the one with $B$’s effectiveness.

Before examining a version of the model with three parties, we briefly discuss endogenous platform placement in the two party model. To do so, return to the Downsian interpretation in Section 4. Now assume voter ideal points are distributed over $\mathbb{R}^2$ according to some distribution $F$ with marginal distributions $F_1$ and $F_2$ (although this argument applies with more than two possible issues). Consider the following setup. First, parties simultaneously choose $x_j \in \mathbb{R}^2$ as policy platforms. The platforms are revealed to both parties, and then the parties play the unique equilibrium in Proposition 2 when $\delta^j > 0$. In any sub-game perfect equilibrium, the two parties should locate at the dimension-by-dimension median ($x_1^\dagger, x_2^\dagger$), where $x_k^\dagger$ is the median of the marginal distribution $F_k$. In other words, because voters will only care about one issue when voting, the placement game inherits the Downsian incentive to locate at the median. Yet these results require two-party competition, and it is likely that with three or more parties the model inherits the equilibrium existence derivative of $\bar{\lambda}^B_2$ with respect to $\beta^B_2$ when $\beta^B_2 = \frac{\delta^A \delta^B}{\delta^A + \delta^B}$.
problems from the one dimensional case as well.

7 An Example with Three Parties and Free Riding

In this section, we use the model to examine why parties campaign on the same issue. As the previous section demonstrates, when there are only two parties and two issues, this type of issue-sharing never occurs as parties always “talk past each other.” Thus, in this section, we consider three parties labelled $A$, $B$, and $C$ who must compete over issues 1 and 2. Party $C$ owns issue 2, i.e., $v^C_2 = 1$, and parties $A$ and $B$ share issue one, i.e., $v^A_1 = \alpha = 1 - v^B_1$, where $\alpha \in \left[\frac{1}{2}, 1\right]$. In other words, parties $A$ and $B$ may be two left parties who share ownership of the redistribution issue (1), with Party $A$ – perhaps the communists – owning redistribution more than $B$ –perhaps, the center left – while Party $C$ represents a right party that owns the fiscal responsibility issue (2).

We characterize an equilibrium in which both Party $A$ and Party $B$ invest resources into issue 1. Denote this equilibrium $\tilde{\lambda}$. By Lemma 1, $\tilde{\lambda}^A = \tilde{\lambda}^B = \tilde{\lambda}^C = 0$ because these issues are dominated issues for the parties, respectively. To simplify the analysis, we make two further assumptions. First, we assume $\delta^j = \delta$ for all parties $j$; in other words, all parties have the same marginal cost of effort. This makes sense if $R^j$ represents monetary resources in a campaign war chest. However, even if this appears restrictive, this assumption does not mean all parties have the same marginal rate of return on investment because the $\beta^j_k$ still vary. Second, we assume that $\delta$ is sufficiently small, where

$$\delta < \frac{\beta^C_2 \sigma_1}{(\sigma_1 + \sigma_2 + \beta^A_1 R^A + \beta^B_1 R^B + \beta^C_2 R^C)^2}. \quad (4)$$

Substantively, the inequality in Eq. 4 implies that parties are encouraged to campaign because raising resources is not too costly. Technically, it guarantees that $D_3 u^C(\lambda) > 0$ for all profiles $\lambda \in \Lambda$, so Party $C$ invests $\tilde{\lambda}^C = R^C$ in equilibrium. This means we only need to pin down $\tilde{\lambda}^A$ and $\tilde{\lambda}^B$.

To do so, first note that $D_1 u^A = D_1 u^B$ if and only if

$$\alpha = \frac{1}{\beta^A_1 + \beta^B_2}. \quad (5)$$

Thus, if $\tilde{\lambda}$ is an equilibrium such that both $A$ and $B$ invest some but not all of their resources, i.e., $\tilde{\lambda}^j \in (0, R^j)$ for $j = A, B$, then the equality in Eq. 5 holds. Furthermore, if both $A$ and $B$ invest some but not all campaign resources in equilibrium $\tilde{\lambda}$, then every $\lambda \in \Lambda$ such that $\lambda^A + \lambda^B = \tilde{\lambda}^A + \tilde{\lambda}^B$, $\lambda^A = \lambda^B = \lambda^C = 0$, and $\lambda^C = R^C$ is an equilibrium. This continuum of equilibria arises because parties $A$ and $B$ have the same marginal benefits and costs of campaigning on issue 1. Thus, they only ensure $\tilde{\lambda}^A + \tilde{\lambda}^B$ is invested in equilibrium.

When the inequality in Eq. 5 does not hold, more interesting equilibrium behavior arises. Without loss of generality, assume $\beta^A_1 \alpha > \beta^B_1 (1 - \alpha)$, which holds when $\beta^A_1 > \beta^B_1$. Then, $D_1 u^A > D_1 u^B$, so one party must always play a corner solution. If Party $A$ does not invest its entire budget ($\tilde{\lambda}^A < R^A$), then $B$ invests no resources ($\tilde{\lambda}^B = 0$). If Party $B$ invests a positive amount ($\tilde{\lambda}^B > 0$), then Party $A$ must invests all its resources ($\tilde{\lambda} = R^A$). That is, when $\delta$ is large, Party $B$ “free rides”
on the campaign of Party A and does not invest. When $\delta$ is small, Party A invests all its resources and only then does Party B invest some positive amount. We present this result formally in the following proposition, which implicitly assumes the assumptions throughout the section.

**Proposition 4** There exists a unique equilibrium $\tilde{\lambda}$ such that $\tilde{\lambda}_A^2 = \tilde{\lambda}_B^2 = \tilde{\lambda}_C^2 = 0$ and $\tilde{\lambda}_C^C = R^C$. Furthermore, there exists bounds $\delta^*$ and $\delta^{**}$ such that $0 < \delta^* < \delta^{**}$ and the following hold.

1. If $\delta \geq \delta^{**}$, then parties A and B invest $\tilde{\lambda}_A^1 = \tilde{\lambda}_B^1 = 0$.
2. If $\delta \in [\delta^*, \delta^{**})$, then $A$ invests $\tilde{\lambda}_A^1 > 0$ and $B$ invests $\tilde{\lambda}_B^1 = 0$.
3. If $\delta \in [0, \delta^*)$, then $A$ invests $\tilde{\lambda}_A^1 = R_A$ and $B$ invests $\tilde{\lambda}_B^1 > 0$.

Substantively, these results provide another reason why parties campaign on the same issue besides attempting to hurt the vote share of a rival (Meguid 2007) or steal ownership (Holian 2004; Sides 2006). The proposition means that distinct parties $j$ and $j'$ both campaign on an issue $k$ when one party exhausts all of its resources on the issue and another party still finds it beneficial to increase the issue’s salience after this initial investment. For instance, suppose $j$ is a radical Green party and $j'$ is a major popular center-left party. The radical Greens “own” an issue about environmental policy in the sense that $\beta_k^j v_k^j > \beta_k^j v_k^{j'}$, or in words, it may be more effective at campaigning on the issue and/or expect a larger vote share on the same issue. Then the Greens exhaust their resources before the center-left begins campaigning on the environment. However, this party is a niche party and may not possess a large amount of electoral resources. When this happens, the center-left potentially spends resources to further increase the salience of the issue because doing so increases the party’s vote share to a considerable degree even after the Greens’ original investment. Thus, the center-left party free rides on the initial investment by the radical Greens.

## 8 Conclusion

We present a new model of electoral competition with issue-ownership and endogenous issue salience, one that closely matches previous substantive theories and empirical results. In the model, parties compete to influence the salience of an issue, which determines the probability that a random voter determines her vote choice according to that issue. Thus, voters do not necessarily have Euclidean preferences, and parties prime voters to care about various issues. In general, parties never emphasize dominated issues, i.e., the issues with their lowest expected vote share, and they almost always best responded to the campaigns of others by highlighting only one issue. In two-party competition, parties talk past each other as the model uncovers Riker’s “dominance principle.” Furthermore, the campaigns of the two parties are highly interconnected, leading to several non-linearities in issue emphasis. Finally, with more than two parties, the model highlights potential free-riding problems in campaigns.

The model synthesizes several new empirical implications. There potentially exists a relationship between the number of parties and the number of issues emphasized in a campaign, and Lemma 2 suggests that as the number of parties decreases so will the number of issues emphasized. In addition, campaigns should consume the most resources when parties have similar, but not identical, campaign capabilities on the issues they own. In addition, Proposition 2 suggests that a party’s optimal issue
emphasis is a highly non-linear function. While this hints at the problems in regression analyses attempting to explain or predict campaign issue emphases, it also reveals a significant benefit of structural estimation to uncover the parameters of interest. In this model, the strategies of the parties can potentially be measured from campaign advertisements or platform texts, and the expected-vote shares from survey data. In this case, we should be able to uncover the relative campaign effectiveness and the natural salience parameters. Theoretically, the model can be expanded to investigate the dynamic of issue-ownership. Campaigning on an issue likely has two consequences. In the short term, it increases the salience of a given issue, but in the long-term it also increases a party’s ownership of the issue. Thus, there is potentially a dynamic trade-off between short- and long-term party electoral strategy.
Appendix

Proof of Proposition 1

Proposition 1 Restated: A Nash Equilibrium exists if any of the following conditions hold.

1. There are two issues, that is, $K = 2$.
2. Parties are effective on the issues they own; that is, for all $j$, there exists an issue $k^j \in \{1, \ldots, K\}$ such that $v^j_{k^j} \geq v^j_{k^j}$ and $\beta_{k^j}^j \geq \beta_{k^j}^j$ for all issues $k'$.

Proof. (1.) When there are only two dimensions, either $V^j$ is constant in $\lambda^j$, that is, $v^j_1 = v^j_2$ or $v^j_1 > v^j_{3-k}$, for some $k = 1, 2$. In the first case, $u^j$ is quasi-concave in $\lambda^j$ because $w^j(\lambda) = v^j_1 - \delta^j(\lambda_1^j + \lambda_2^j)$. In the second case, investing any resources into issue 3 $- k$ is strictly dominated. In addition, $V^j$ is then strictly concave in $\lambda_k^j$. Thus, $u^j$ is concave in non-dominated strategies, and the DGF Theorem guarantees existence because $u^j$ is continuous and $\lambda^j$ is convex and compact.

(2.) Consider some profile $\lambda^{-j}$. We claim that the set of $j$’s best responses to $\lambda^{-j}$—label it $B^j(\lambda^{-j})$—is convex, which means that the DGF Theorem implies that an equilibrium exists, as in the previous proof. To see this, let $M^j \subseteq \{1, \ldots, K\}$ denote the set of issues $k$ such that $v^j_k \geq v^j_{k^j}$ and $\beta^j_k \geq \beta^j_{k^j}$ for all $k' = 1, \ldots, K$. This set is non-empty by assumption, so define $k^* = \min M^j$, which exists. Throughout, we assume there exists an issue $k \neq k^*$ such that $v^j_{k^*} > v^j_{k^j}$, that is, $M^j \not\subseteq \{1, \ldots, K\}$.

If $M^j = \{1, \ldots, K\}$, then $V^j$ is constant in $\lambda^j$. This would mean $B^j(\lambda^{-j})$ consists of either the zero vector ($\delta^j > 0$) or the entire strategy space $\lambda^j$ ($\delta^j = 0$), both of which are convex.

We now establish three preliminary claims. First note that $B^j(\lambda^{-j}) \subseteq \{\lambda^j \mid \lambda^j_l = 0, \forall l \notin M^j\}$ because for any $k \in M^j$, any $l \in \{1, \ldots, K\} \setminus M^j$, and any profile $\lambda$, $D_k u^j(\lambda) > D_l u^j(\lambda)$ by Lemma 2. Second, note that $B^j(\lambda^{-j})$ is non-empty because $w^j(\cdot, \lambda^{-j})$ is a continuous function and $\Lambda^j$ is a convex and compact set. Now define the profile $\tilde{\alpha}^j \in \Lambda^j$ as follows:

$$\tilde{\alpha}^j = \arg \max_{\lambda^j : \lambda^j_k = 0, k \neq k^*} u^j(\lambda^j, \lambda^{-j}).$$

The profile $\tilde{\alpha}^j$ exists and is unique. To see this, note that $[0, R^j]$ is convex and compact. In addition, $u^j$ is continuous and strictly concave in $\lambda^j_k$, because the second derivative of $u^j$ with respect to $\lambda^j_k$, is

$$-2 \left(\beta^j_{k^*}\right)^2 \frac{\sum_k (v^j_{k^*} - v^j_k) \left[\sigma_{k'j} + \sum_{j'} \beta^j_{k'} \lambda^j_{k'}\right]}{\left(\sum_k \left[\sigma_k + \sum_{j'} \beta^j_{k} \lambda^j_{k'}\right]\right)^3},$$

which is strictly less than zero because $\sigma_k > 0$ and $v^j_{k^*} - v^j_k \geq 0$ for all $k$ and is positive for at least one $k \neq k^*$ by assumption. Third, for all $\lambda^j \in S$, where

$$S = \left\{\lambda^j \mid \sum_{k \in M^j} \lambda^j_k = \sum_k \tilde{\alpha}^j_k \text{ and } \lambda^j_l = 0, \forall l \notin M^j\right\},$$

$u^j(\lambda^j, \lambda^{-j}) = u^j(\tilde{\alpha}^j, \lambda^{-j})$ because $v^j_k = v^j_{k^*}$ and $\beta^j_k = \beta^j_{k^*}$ for all $k, k' \in M^j$.
We now demonstrate that $B^j(\lambda^{-j}) = S$. To do so, consider some $\lambda^j \in B^j(\lambda^{-j})$, which exists by the second claim, and suppose $\lambda^j \notin S$. Because $\lambda^j$ is a best response, $\lambda^j_l = 0$ for all $l \notin M^j$ by the first claim above. This means $\sum_{k \in M^j} \lambda^j_k \neq \sum_k \bar{\alpha}^j_k$ by the third claim. Without loss of generality assume $\sum_{k \in M^j} \lambda^j_k < \sum_k \bar{\alpha}^j_k$. Then $\sum_k \bar{\alpha}^j_k > 0$, which means $D_k \cdot u^j(\bar{\alpha}^j, \lambda^{-j}) \geq 0$ is the necessary first-order condition for maximization procedure above. Then we have

$$D_k \cdot u^j(\lambda) = \beta^j_k \cdot \frac{v^j_{k^*} - V^j(\lambda)}{\sum \sigma_k + \sum_{k'} \beta^j_k \lambda^j_k} - \delta^j$$

$$> \beta^j_k \cdot \frac{v^j_{k^*} - V^j(\bar{\alpha}^j, \lambda^{-j})}{\sum \sigma_k + \sum_{k'} \beta^j_k \lambda^j_k} - \delta^j$$

$$> \beta^j_k \cdot \frac{v^j_{k^*} - V^j(\bar{\alpha}^j, \lambda^{-j})}{\sum \bar{\alpha}^j_k + \sum \sigma_k + \sum_{j' \neq j} \beta^j_k \lambda^j_k} - \delta^j$$

$$= D_k \cdot u^j(\bar{\alpha}^j, \lambda^{-j})$$

$$\geq 0,$$

where the inequalities follow because there exists some $k$ such that $v^j_{k^*} < v^j_{k^*}$, and $\sum_{k \in M^j} \lambda^j_k < \sum_k \bar{\alpha}^j_k$. So $j$ has a profitable deviation by investing more into issue $k^*$, which means $\lambda^j$ cannot be a best response, a contradiction.

Now consider some $\lambda^j \in S$ and suppose there exists some $\mu^j$ such that $u^j(\mu^j, \lambda^{-j}) > u^j(\lambda^j, \lambda^{-j})$. Then let $\bar{\mu}^j$ denote the profile such that $j$ allocates the total spent resources in profile $\mu^j$ to issue $k^*$ and allocates zero resources to all other issues. That is $\bar{\mu}^j = (0, \ldots, \sum_k \mu^j_k, \ldots, 0)$, where $\sum_k \mu^j_k$ is the $k^*$th entry. So we now have

$$u^j(\bar{\mu}^j, \lambda^{-j}) \geq u^j(\mu^j, \lambda^{-j}) > u^j(\lambda^j, \lambda^{-j}) = u^j(\bar{\alpha}^j, \lambda^{-j}),$$

but this means that $\bar{\alpha}^j$ does not solve the maximization procedure above, a contradiction. \qed

**Proof of Proposition 2**

To prove the proposition, we first use Proposition 1 to guarantee existence. Second, we characterize the four possible equilibria of the model, which include a unique profile under which both parties invest positive resources and three “corner” profiles under which at least one party invests no resources. We then show that when one of these equilibria exist, the others do not. Throughout, we maintain assumptions (1A)-(1C).

**Result 1** If $(\lambda^A, \lambda^B)$ is an equilibrium such that $\lambda^A_1 > 0$ and $\lambda^B_2 > 0$, then $\lambda^A = (\lambda^A_1, 0)$ and $\lambda^B = (\lambda^B_2, 0)$.

**Proof.** Consider an equilibrium $(\lambda^A, \lambda^B)$ such that $\lambda^A_1 > 0$ and $\lambda^B_2 > 0$. Then $\lambda^A_2 = \lambda^B_1 = 0$. In addition, $D_1 u^A = 0$ and $D_2 u^B = 0$. Using Eq. 3 and solving for $(\lambda^A_1, \lambda^B_2)$ gives us the polynomials in the proposition. The solutions are maximizers because the second derivative of $u^A$ with respect
\[ \lambda_A^1, \text{ denoted } D_1^2 u^A, \text{ is} \]
\[ D_1^2 u^A = -2 \left( \beta_1^A \right)^2 \frac{(v_1^A - v_2^A)(\beta_1^A \lambda_1^A + \sigma_1)}{(\sigma_1 + \beta_1^A \lambda_1^A + \sigma_2 \beta_2^B \lambda_2^B)^3}, \]
which is strictly less than zero by (A1). The same holds for \( D_2^2 u^B \).

Define the following strategies:
\[ \hat{\lambda}_1^A = \frac{\sqrt{(v_1^A - v_2^A)(\beta_1^A \lambda_1^A + \sigma_1)}}{(\sigma_1 + \beta_1^A \lambda_1^A + \sigma_2 \beta_2^B \lambda_2^B)^2} \]
\[ \hat{\lambda}_2^B = \frac{\sqrt{(v_1^A - v_2^A)(\beta_2^B \lambda_2^B + \sigma_2)}}{(\sigma_1 + \beta_1^A \lambda_1^A + \sigma_2 \beta_2^B \lambda_2^B)^2} \]

**Result 2** Party A’s unique best response to Party B’s investment \((0, 0)\) is \( (\max \left\{ 0, \hat{\lambda}_1^A \right\}, 0) \), and Party B’s unique best response to Party A’s investment \((0, 0)\) is \( (0, \max \left\{ 0, \hat{\lambda}_2^B \right\}) \).

**Proof.** We prove the result for Party A. Assume Party B invests \((0, 0)\). Then the necessary and sufficient first order condition \( D_1 u^A = 0 \) is equivalent to A investing \( \hat{\lambda}_1^A \). This best response is unique because \( u^A \) is strictly concave in \( \lambda_1^A \). If \( \hat{\lambda}_1^A \) is less than zero, strict concavity guarantees that \( u^A \) is decreasing in \( \lambda_1^A \). In this case, A’s best response is to invest 0 in the first issue.

**Result 3** Party A’s unique best response to Party B’s investment \((0, \hat{\lambda}_2^B)\) when \( \hat{\lambda}_2^B > 0 \) is \((0, 0)\) if and only if \( \hat{\lambda}_1^A < 0 \), and Party B’s unique best response to Party A’s investment \((\hat{\lambda}_1^A, 0)\) when \( \hat{\lambda}_1^A > 0 \) is \((0, 0)\) if and only if \( \hat{\lambda}_2^B < 0 \).

**Proof.** We prove the result for Party B. Fix A’s strategy at \((\hat{\lambda}_1^A, 0)\) when \( \hat{\lambda}_1^A > 0 \). For Party B to invest 0 in issue 2, we must guarantee \( D_2 u^B < 0 \). To see this, note that when \( \hat{\lambda}_1^A > 0 \), we have
\[ D_2 u^B < 0 \iff -\delta_B + \beta_2^B \frac{\sqrt{(v_1^A - v_2^A)(\beta_1^A \lambda_1^A + \sigma_1)}}{\beta_1^A \lambda_1^A + \sigma_2 \beta_2^B \lambda_2^B} < 0 \]
\[ \iff \sigma_2 > \frac{(\beta_2^B)^2 \beta_1^A \lambda_1^A (v_1^A - v_2^A)}{(\beta_1^A \sigma_1 + \beta_2^B \lambda_2^B)^2}, \]
which is equivalent to \( \hat{\lambda}_2^B < 0 \).

**Result 4** There are four potential equilibria, which are as follows:
1. \( (\hat{\lambda}_1^A, 0), (0, \hat{\lambda}_2^B) \)
2. \( (\hat{\lambda}_1^A, 0), (0, 0) \)
3. \( (0, 0), (\hat{\lambda}_2^B, 0) \)
4. \( (0, 0), (0, 0) \)
Proof. (1) This equilibrium exists if and only if $\lambda_1^A \geq 0$ and $\lambda_2^B \geq 0$ by Result 1. (2) This equilibrium exists if and only if $\lambda_1^A > 0$ and $\lambda_2^B \leq 0$ by Results 2 and 3. (3) This equilibrium exists if and only if $\lambda_2^B > 0$ and $\lambda_2^A \leq 0$ by Results 2 and 3. (4) This equilibrium exists in all other cases by Proposition 1 and the three previous statements. □

We now prove the uniqueness of each of the potential equilibria.

Uniqueness of (1). Suppose $((\hat{\lambda}_1^A, 0), (0, \hat{\lambda}_2^B))$ is an equilibrium, and suppose $\lambda$ is another equilibrium such that $\lambda \neq ((\hat{\lambda}_1^A, 0), (0, \hat{\lambda}_2^B))$. By Lemma 1, $\lambda_2^A = \lambda_2^B = 0$. By Result 1, there is an unique interior equilibrium, so either $\lambda_1^A = 0$ or $\lambda_2^B = 0$. Without loss of generality, consider the case in which $\lambda_1^A = 0$. If $\lambda_2^B > 0$, then Result 2 implies $\lambda_2^B = \hat{\lambda}_2^B$. Result 3 then implies $D_1 u^A > 0$, a contradiction because $A$ can deviate by investing some small amount into issue 1. If $\lambda_2^B = 0$, we must rule out the case in which $((0, 0), (0, 0))$ is an equilibrium. To do so, without loss of generality suppose $\sigma_2 \geq \sigma_1$. Then

$$D_1 u^A((0, 0), (0, 0)) = \beta_1^A \frac{(v_1^A - v_2^A)\sigma_2 - \delta^A}{(\sigma_1 + \sigma_2)^2} \geq \beta_1^A \frac{(v_1^A - v_2^A)\sigma_2 + \beta_2^B \lambda_2^B}{(\sigma_1 + \sigma_2 + \beta_2^B \lambda_2^B)^2} - \delta^A$$

$$> \beta_1^A \frac{(v_1^A - v_2^A)(\sigma_2 + \beta_2^B \lambda_2^B)}{(\sigma_1 + \beta_1^A \lambda_1^A + \sigma_2 + \beta_2^B \lambda_2^B)^2} - \delta^A$$

$$= D_1 u^A((\hat{\lambda}_1^A, 0), (0, \hat{\lambda}_2^B)) = 0,$$

where the first inequality follows because $\frac{(v_1^A - v_2^A)(\sigma_2 + x)}{(\sigma_1 + \sigma_2 + x)^2}$ is (weakly) decreasing in $x \geq 0$ when $\sigma_2 \geq \sigma_1$, the second because $D_1^2 u^A < 0$, and the final equality because $((\hat{\lambda}_1^A, 0), (0, \hat{\lambda}_2^B))$. But $D_1 u^A((0, 0), (0, 0)) > 0$ implies $A$ has a profitable deviation. □

Uniqueness of (2) and (3). We prove the uniqueness of the second equilibrium, and an identical argument proves the uniqueness of the third. Suppose the equilibrium $((\hat{\lambda}_1^A, 0), (0, 0))$ exists and suppose $\lambda$ is an equilibrium such that $\lambda \neq ((\hat{\lambda}_1^A, 0), (0, 0))$. By Lemma 1, $\lambda_2^A = \lambda_1^B = 0$. By Result 3, the equilibrium in which both candidates invest positively does not exist so either $\lambda_1^A = 0$ or $\lambda_2^B = 0$. If $\lambda_2^B = 0$, then $\lambda = ((\hat{\lambda}_1^A, 0), (0, 0))$ by Result 3, a contradiction. So $\lambda_1^A = 0$ and $\lambda_2^B > 0$, and $\lambda_2^B = \hat{\lambda}_2^B$ by Result 2. Then either $\sigma_1 \geq \sigma_2$ or $\sigma_2 \geq \sigma_1$. Without loss of generality, assume the latter. Then

$$D_1 u^A((\hat{\lambda}_1^A, 0), (0, 0)) = 0 \implies D_1 u^A((0, 0), (0, 0)) > 0$$

$$\implies D_1 u^A((0, 0), (0, \hat{\lambda}_2^B)) > 0,$$

where the second implication follows from the same logic in the previous proof, i.e., $D_1 u^A((0, 0), (0, \lambda_2^B))$ is increasing in $\lambda_2^B$, when $\sigma_2 \geq \sigma_1$. But this means $\lambda$ cannot be an equilibrium because $A$ has a profitable deviation by investing some small amount in issue 1. □

Uniqueness of (4). Suppose the equilibrium $((0, 0), (0, 0))$ exists. The equilibrium in which only one
candidate invests positive resources cannot exist because $D_1 u^A(0,0) \geq 0$ and $u^j$ is concave in $j$'s non-dominated issue. Finally, because $D_1 u^A(0,0) \leq 0$, the string of inequalities in the proof that demonstrates the uniqueness of the interior equilibrium implies that $A$'s (B's) best response to $B$'s (A's) investment of positive resources is to invest zero resources as well.

\[\square\]

**Proof of Proposition 4**

Let $\hat{\lambda}$ be an equilibrium. Note that one indeed exists by Proposition 1. First, recall that $\hat{\lambda}^A = 0$ by Lemma 1.

Second, $\hat{\lambda}^C = 1$ because

\[
D_2 u^C(\hat{\lambda}) = \beta^C \frac{\sigma_1 + \beta_1^A \hat{\lambda}^A + \beta_1^B \hat{\lambda}^B}{(\sigma_1 + \beta_1^A \hat{\lambda}^A + \beta_1^B \hat{\lambda}^B + \sigma_2 + \beta_2^C \hat{\lambda}^C)^2} - \delta \\
\geq \beta^C \frac{\sigma_1}{(\sigma_1 + \beta_1^A \hat{\lambda}^A + \beta_1^B \hat{\lambda}^B + \sigma_2 + \beta_2^C \hat{\lambda}^C)^2} - \delta \\
> 0,
\]

where the last inequality follows by the assumption that $\delta$ is sufficiently small. Thus, $C$ always increases her payoff by investing more, so $\hat{\lambda}^C = R^C$.

Third, we pin down equilibrium values of $\hat{\lambda}^A$ and $\hat{\lambda}^B$. To do so, we establish two relationships between $\hat{\lambda}^A$ and $\hat{\lambda}^B$. By straightforward differentiation, we have

\[
\hat{\lambda}^A \leq 0 \iff D_1 u^B(\hat{\lambda}) \leq 0 \quad \Rightarrow \quad D_1 u^B(\hat{\lambda}) < 0
\]

and

\[
\hat{\lambda}^B > 0 \iff D_1 u^B(\hat{\lambda}) \geq 0 \quad \Rightarrow \quad D_1 u^A(\hat{\lambda}) > 0
\]

So we must find two cut-points. First, $\delta \geq \delta^{**}$ needs to imply $A$ never invests, i.e. $D_1 u^A(\hat{\lambda}) \leq 0$ when $\hat{\lambda}^B = 0$. By the implication above, $B$ will have no incentive to invest. Second, $\delta < \delta^{**}$ needs to imply $B$ invests some positive share, i.e. $D_1 u^B(\hat{\lambda}) \leq 0$ when $\hat{\lambda}^A > 0$. But then $D_1 u^B(\hat{\lambda}) \leq 0$ implies $D_1 u^A(\hat{\lambda}) > 0$, which can only happen when $\hat{\lambda}^A = R_A$.

To compute $\delta^{**}$, fix $\hat{\lambda}^B = 0 = \hat{\lambda}^A$. We have

\[
D_1 u^A(\hat{\lambda}) \leq 0 \iff \delta \geq \frac{\alpha \beta_1^A (\beta_2^C R^C + \sigma_2)}{(\beta_2^C R^C + \sigma_1 + \sigma_2)^2} = \delta^{**}
\]

To compute $\delta^{*}$, fix $\hat{\lambda}^A = R^A$ and $\hat{\lambda}^B = 0$. We now have

\[
D_1 u^B(\hat{\lambda}) \leq 0 \iff \delta \geq \frac{(1 - \alpha) \beta_1^B (\beta_2^C R^C + \sigma_2)}{(\beta_1^A R^A + \sigma_1 + \beta_2^C R^C + \sigma_2)^2} = \delta^{*}.
\]
Finally, \( \delta^* < \delta^{**} \) because \((1 - \alpha)\beta_1^B < \alpha \beta_1^A \) by assumption.

Using necessary and sufficient first order conditions, we can write strategies as follows:

\[
\tilde{\lambda}_1^A = \begin{cases} 
\max\{x^A, R^A\} & \text{if } \delta < \delta^{**} \\
0 & \text{if } \delta \geq \delta^{**} 
\end{cases},
\]

where

\[
x^A = \frac{\sqrt{\alpha \left(\beta_1^A\right)^3 \delta (\beta_2^C R^C + \sigma_2) - \beta_1^A \delta (\beta_2^C R^C + \sigma_2 + \sigma_1)}}{\left(\beta_1^A\right) \delta},
\]

and

\[
\tilde{\lambda}_1^B = \begin{cases} 
\max\{x^A, R^A\} & \text{if } \delta < \delta^{*} \\
0 & \text{if } \delta \geq \delta^{*} 
\end{cases},
\]

where

\[
x^B = \frac{\sqrt{(1 - \alpha) \left(\beta_1^B\right)^3 \delta (\beta_2^C R^C + \sigma_2) - \beta_1^B \delta (\beta_2^C R^C + \sigma_2 + \beta_1^A R^A \sigma_1)}}{\left(\beta_1^B\right) \delta}.
\]

References


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